An Investigation Into the Use of Orthogonal Winding in Loosely Coupled Link for Improving Power Transfer Efficiency Under Coil Misalignment

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Abstract—It is sometimes unavoidable to use loosely coupled coils in applications, like biomedical devices, for transferring electric energy wirelessly. However, coil misalignment causes degradation of the power transfer efficiency. It is well known that the power transfer efficiency of classical parallel coils is primarily determined by the quality factors of the coils and the coupling coefficient between the coils, and is maximized by choosing an optimal turns-ratio between the coils. Changing the number of turns of the coils cannot effectively overcome such misalignment effect. This paper presents a structure that comprises two orthogonally placed windings for lessening the variation of the coupling coefficient due to the coil misalignment. An output current summing technique that keeps the windings concurrently energized and combines the output currents of the windings will be studied. A canonical model will also be derived to describe the interactions between the coils. An experimental prototype has been built and evaluated on a test bed, which allows different degrees of lateral and angular misalignments. Results reveal that the proposed structure can effectively increase the minimum efficiency zone, allowing more lateral and angular misalignments. These investigations lay the foundation for future understanding more complex loosely coupled winding structures.

Index Terms—Coil misalignments, winding structure, wireless inductive link, wireless power transfer.

I. INTRODUCTION

WIRELESS inductive links have been widely used in many applications, such as cochlear implants [1], retinal prostheses [2]–[4], battery chargers [5], transportation [6], [7], and induction heating [8]. They are generally composed of a transmitter, an end-use device, and two coils with one coil in the transmitter and another one in the end-use device. As some applications, such as biomedical applications, do not allow using any ferromagnetic core, the two coils are loosely coupled. The transmitter and receiver coils are typically placed in parallel. Electric energy is transmitted from the transmitting coil to the receiving coil through alternating magnetic fields. The system is optimized toward maximum power transfer efficiency [9]–[12]. Although many improved transmitter and receiver designs have emerged, the link efficiency is still determined by a fundamental “bottleneck”—fluctuations in the power transfer and link efficiency due to the coil misalignment. When the coils are coaxially orientated, the coils are well coupled and thus the link efficiency is maximal. However, if the two coils are misaligned, the magnetic coupling and the overall link efficiency will impair.

As discussed in [9], the maximum power transfer efficiency of parallel placed coils is dependent on the coil quality factors, as defined in (E10), and the coupling coefficient between the two coils, as defined in (35) and (36). The coupling coefficient gives measure of the degree of magnetic coupling, determined by the coil sizes and geometric spacing. When the coils are coaxially orientated, their coupling is the strongest. However, the coils could be misaligned axially, laterally, and angularly in practical situations, impairing their magnetic coupling [13]. Take retinal prosthesis utilizing wireless telemetry for power transfer as an example, axial and lateral misalignment would occur upon displacement of the pair of glasses [3]. More importantly, impairment of magnetic coupling, due to the coil misalignment [13]–[15], can lead to reduction of power efficiency. There is a rule of thumb in maximizing misalignment tolerance for disk-shaped primary coil. The primary coil diameter should be larger than that of secondary coil and equal to twice the distance between two coils [14], [16]. However, there are concerns with this design approach. First, the volt–amp rating of the transmitter is nonoptimal, as the variation of the coupling coefficient is large and a large portion of magnetic flux generated by the transmitter is uncoupled to the secondary coil, no matter under aligned or misaligned condition. Second, a large portion of magnetic flux is transmitted to free space. Third, the coils have to be oversized in order to improve the misalignment tolerance.

There are structures having multiple windings in the transmitter and/or receiver [5], [17]–[25] for different operational targets. The methods discussed in [5] and [17] have multiple...
displaced parallel transmitter coils to establish a near-uniform perpendicular flux around the receiver coil. Due to the confined flux direction, the coupling method is specifically used to tackle lateral misalignment. The 3-D structure discussed in [18] and [19] offers an omnidirectional coupling. Three windings in the receiver are placed orthogonally. Each winding is resonated by a parallel capacitor and its output is rectified by a diode circuit. Then, the rectified outputs are paralleled [26, 27]. As discussed in [19], since the three voltage outputs are in parallel, it can only allow a narrow operation region to simultaneously energize all windings. Thus, the worst-case scenario is that only one of the three windings in the structure delivers output power. It is effectively a single-coil-to-single-coil coupling, and thus cannot guarantee to operate at the achievable maximum power transfer. The methods discussed in [20]–[22] utilize resonator coils to maximize power transfer in a relatively longer distance under an aligned condition. The added extra winding in [23] functions as a data coil for communication.

To attain a high-level magnetic coupling under misaligned conditions, this paper provides a new perspective and detailed investigation into the use of parallel and orthogonal windings in the receiving coil set for loosely coupled link. An output current summing technique, based on using an inductor to form a current source at the output of the rectifier, is proposed to broaden the range of possible misalignments where the secondary coils are energized. For the sake of simplicity in the analysis and illustration, two orthogonally placed windings, instead of three as discussed in [18] and [19], are studied.

The performance of an experimental prototype with two coil sets, including the classical parallel coil structure and the proposed structure, will be evaluated. The experimental results are favorably compared to the theoretical predictions. The conclusions follow in the last section.

II. Motivation

The wireless inductive link with square-shaped loosely coupled coils is illustrated in Fig. 1. The power transfer efficiency versus the number of turns in the receiving coil will be studied further on in this paper theoretically and experimentally. The parameters of the 16-turn transmitting coil used in the study are given in Table I. Five receiving coils having 6, 12, 18, 24, and 36 turns, respectively, are fabricated, in order to study the transfer efficiency, which is defined as the ratio between the load power $P_{\text{out}}$ and the power supplied to the transmitting coil $P_{\text{in}}$. Fig. 2 shows the results when the coils are under three testing cases—aligned and two misaligned conditions. For the sake of comparison, the load resistor and the resonant capacitors for the transmitting and receiving coils, respectively, are designed such that maximum power transfer efficiency occurs in all cases. The theoretical analysis is based on the method described in [9]. Several nonideal characteristics, such as the power loss in the diode-bridge rectifier, winding resistance, etc., are taken into account. Results reveal that the efficiency is increased when the number of turns is increased from 6 turns to 12 turns. Then, it becomes fairly constant, even if the number of turns is increased to 36 turns. It is mainly because the ac resistance of the coils increases with the increase in the number of turns, due to the proximity effect [28]. Such resistive effect becomes dominant and gives an adverse effect on the efficiency, even if the flux linkage between the transmitting and receiving coils is also increased with the increased number of turns. The study concludes that the maximum efficiency is primarily determined by the quality factors of the coils and the coupling coefficient between the coils. The efficiency variations versus the number of turns in the three testing cases are similar, demonstrating that variation of the transfer efficiency, due to coil misalignment, cannot be overcome effectively by simply changing the number of turns in the receiving coil.

A T-shaped structure with two orthogonally placed windings, as shown in Fig. 3, is investigated. It is composed of two windings, namely parallel and orthogonal windings which are similar to the cross-shaped structure presented in [29]. The
Fig. 3. Proposed winding structure with a parallel winding and an orthogonal winding.

Fig. 4. Mutual inductance of two filaments.

T-shaped structure makes the parallel winding deliver more power as the parallel winding is placed closer to the transmitting coil for the same axial separation between the two coils. The parallel winding has the same number of turns as the optimal number of turns in the parallel coil structure, i.e., 12 turns in the case study. The orthogonal winding is used to assist with increasing the flux linkage between the coils under the misaligned condition. Extensive mathematical treatment and investigations into the power transfer phenomenon of the T-shaped structure will be conducted in the following sections.

III. MUTUALINDUCTANCE BETWEEN COILS

A. Formulation of the Mutual Inductance

In order to study the effect of misalignment on affecting the power transfer efficiency, the following investigations start with calculating the mutual inductance and magnetic field distribution between the coils. The mutual inductance is defined as the number of flux linkages with the secondary coil due to unit current in primary coil. It is determined by the double integral Neumann formula [30]:

\[
M = \frac{\mu_0}{4\pi} \int_T \int_R \frac{dl_T \cdot dl_R}{r_{TR}} \cos \varepsilon \tag{1}
\]

where \(dl_T\) and \(dl_R\) are the infinitesimal segments of the transmitting coil and receiving coil, respectively, and \(r_{TR}\) and \(\varepsilon\) are the distance and angle between the two segments, respectively. Equation (1) is applicable for different shapes of coils.

The mutual inductance \(M_f\) between two filaments of lengths \(l\) and \(m\), respectively, as shown in Fig. 4 is [30]

\[
M_f = \text{sgn}(\hat{I}_1 \cdot \hat{I}_2)0.001 \cos \varepsilon
\]

\[
= \begin{cases} 
2 \left[ \left( \frac{\mu + l}{R_1 + R_2} \right) \tanh^{-1} \frac{m}{R_1 + R_2} + \left( \frac{\nu + m}{R_3 + R_4} \right) \tanh^{-1} \frac{l}{R_3 + R_4} \right] - \frac{\Omega d}{\sin \varepsilon} & \text{if } \hat{I}_1 \cdot \hat{I}_2 > 0 \\
-2 \left[ \left( \frac{\mu + l}{R_1 + R_2} \right) \tanh^{-1} \frac{m}{R_1 + R_2} + \left( \frac{\nu + m}{R_3 + R_4} \right) \tanh^{-1} \frac{l}{R_3 + R_4} \right] + \frac{\Omega d}{\sin \varepsilon} & \text{if } \hat{I}_1 \cdot \hat{I}_2 < 0 
\end{cases}
\]  

where \(\hat{I}_1\) and \(\hat{I}_2\) are the current unit vectors of filament AB and CD, respectively, \(\text{sgn}(\hat{I}_1 \cdot \hat{I}_2)\) is the sign of the dot product between \(\hat{I}_1\) and \(\hat{I}_2\). The mathematical expressions of the parameters are given in Appendix A.

In the following discussion, the coils shown in Fig. 1 are illustrated. Both the transmitting and receiving coils have four sets of wires having the same spatial orientation. They are sets I–IV in the transmitting coil, and sets i–iv in the receiving coil. The mutual inductance \(M_{12}\) between the transmitting coil and the receiving coil is calculated by summing up the mutual inductances of each set of wires in the transmitting coil with all other sets in the receiving coil. Thus

\[
M_{12} = N_1 N_2 \sum_{i=1}^{IV} \sum_{j=1}^{iv} M_{f,ij} \tag{3}
\]

where \(N_1\) and \(N_2\) are numbers of turns of the transmitting coil and receiving coil, respectively, and \(M_{f,ij}\) is the mutual inductance between a wire in the \(i\)th set in the transmitting coil and a wire in the \(j\)th set in the receiving coil.

B. Measurement of the Mutual Inductance Between Coils

The mutual inductance between two coupled coils is determined by measuring the self-inductance of each coil, and the total inductances when the two coils are connected in series. As illustrated in Fig. 5, there are two possible modes of connection: 1) series-aiding mode (SAM); and 2) series-opposing mode (SOM). In SAM, the magnetic fields generated by the coils are aided each other. The total inductance \(L_{SAM}\) is

\[
L_{SAM} = L_1 + L_2 + 2M_{12} \tag{4}
\]

Fig. 5. Connections of the two coils for measuring mutual inductance. (a) SAM. (b) SOM.
where $L_1$ and $L_2$ are the self-inductances of the transmitting and receiving coils, respectively, and $M_{12}$ is the mutual inductance between the two coils.

Conversely, in SOM, the magnetic fields generated by the coils are opposed to each other. The total inductance $L_{\text{SOM}}$ is

$$L_{\text{SOM}} = L_1 + L_2 - 2M_{12}.$$  \hfill (5)
By using (4) and (5), \( M_{12} \) can be determined by the measured values of \( L_1 \) and \( L_2 \), \( L_{\text{SAM}} \), and \( L_{\text{SOM}} \) using the formulas of

\[
M_{12} = \frac{1}{2}(L_{\text{SAM}} - L_1 - L_2)
\]

or

\[
M_{12} = \frac{1}{2}(L_1 + L_2 - L_{\text{SOM}}).
\]

C. Mutual Inductance Under Lateral and Angular Misalignments

When the centers of the transmitting and receiving coils are aligned, their mutual inductance is maximal. However, it is practically unavoidable to have coil misalignments. In this section, variations of their mutual inductance under lateral and angular misalignments are studied. Fig. 1 illustrates a general case where the coils are misaligned. Let \( d \) be the distance between the centers of the two coils under perfectly aligned condition, \( \Delta \) be the lateral displacement between the centers of the two coils under lateral misalignment, and \( \theta \) be the angle between the planes of the two coils under angular misalignment. Based on the parameters given in Tables I and II, Fig. 6(a) and (b) shows the calculated and measured mutual inductance \( M_{12} \) under lateral misalignment \( \Delta \) with \( \theta = 0 \) and angular misalignment \( \theta \) with \( \Delta = 0\% \), respectively. The calculations are based on the equations given in (1)–(3), while the measurements are conducted by the method described in Section III-B.

When the two coils are perfectly aligned, i.e., \( \Delta \) and \( \theta \) are both zero, \( M_{12} \) is the highest. \( M_{12} \) reduces with an increase in any misalignment. As shown in Fig. 6, under a lateral misalignment of 50% relative to the length of the transmitting coil, \( M_{12} \) is reduced by 50%. Under an angular misalignment of 45°, \( M_{12} \) is reduced by 20%. The value of \( M_{12} \) will decrease more rapidly as the angular misalignment is further increased. The coupling becomes zero at the maximum misalignment. With the help of Fig. 7(a), the reduction in \( M_{12} \) can be visualized by considering the flux linkage between the two coils under various misaligned conditions. For the sake of simplicity, only lateral misalignment is illustrated. Assume that the position of the transmitting coil is fixed and the position of the receiving coil is movable. The flux linkage between the two coils is maximal when the plane of the receiving coil is perpendicular to the magnetic flux. At position “1,” the orientation of the receiving coil is the best. As the receiving coil is displaced from position “1” to “4,” the magnetic flux becomes not perpendicular to the plane of the transmitting coil anymore, but emanates outward. Thus, the flux linkage is diminished, as the coil is approaching to the edge of transmitting coil. Fig. 7(b) shows the desired orientation of the receiving coil at the four positions. At position “1,” the receiving coil should be placed in parallel with the transmitting coil. As the position is moving toward the edge of the transmitting coil, the receiving coil should be rotated in order to pick up maximum magnetic flux. At position “4,” it should be placed nearly orthogonal to the transmitting coil. Therefore, parallel coil can pick up more flux near the center of the transmitting coil, while orthogonal coil can pick up more flux near the edge of the transmitting coil.

Fig. 7. Visualization of the flux linkage under the misaligned condition. (a) Illustration of the flux linkage under lateral misalignments. (b) Desired orientation of the receiving coil for maximum flux linkage.

D. Mutual Inductances of the Proposed Structure

Based on the previous observation, the T-shape receiving coil structure shown in Fig. 3 is investigated to tackle the reduction of the mutual inductance under coil misalignments. One end of the added orthogonal winding is aligned on the same plane of the parallel winding, so that the space between the transmitting coil and the proposed receiving coil is the same as the parallel-coil structure in Fig. 1. Without loss of generality, the two windings are assumed to have the same dimensions. Let \( M_{12} \) be the mutual inductance between the transmitting coil and the parallel winding, and \( M_{13} \) be the mutual inductance between the transmitting coil and the orthogonal winding. It should be noted that there is no inductive coupling between the parallel winding and the orthogonal winding as the two windings are spatially orthogonal. Fig. 8(a) and (b) shows the profiles of \( M_{12} \) and \( M_{13} \) under lateral misalignment with \( \theta = 0 \) and angular misalignment with \( \Delta = 0\% \), respectively. The number of turns of the orthogonal winding is twice that of the parallel winding. When the receiving coil is perfectly aligned with the transmitting coil, \( M_{12} \) is maximized and \( M_{13} \) is zero. When the receiving coil is displaced, \( M_{12} \) drops but \( M_{13} \) increases. It should be noted that the polarity of the orthogonal windings is interchanged when \( \Delta \) is negative.
IV. MODELING AND CIRCUIT IMPLEMENTATION

In order to maximize the utilization of the two windings in the power transfer, they are energized concurrently by using a current summing circuit. Such structure is studied by first modeling the windings with a proposed canonical transformer model and then applying it to develop a circuit model to study the overall power transfer phenomena.

A. Derivation of Canonical Transformer Model

A canonical transformer model that describes the interaction between the primary and secondary windings is derived. Fig. 9(a) shows the classical transformer model, in which the numerals “1” and “2” appeared in the subscripts denote the primary and secondary sides of the transformer, respectively. \( L \) and \( r \) are the self-inductance and resistance of the respective winding, respectively. The two windings have the turns ratio \( n_{12} \) and mutual inductance \( M_{12} \). Based on Fig. 9(a), the following Laplace-transformed equations can be derived:

\[
\begin{bmatrix}
V_1(s) \\
V_2(s)
\end{bmatrix} = \begin{bmatrix}
sL_1 + r_1 & -sM_{12} \\
M_{12} & sL_2 + r_2
\end{bmatrix} \begin{bmatrix}
I_1(s) \\
I_2(s)
\end{bmatrix}.
\]

(8)

Equation (8) is rearranged to derive a canonical model as follows:

\[
V_1(s) = \left[ Z_1(s) + \frac{s n_{12}^2 M_{12} Z_2(s)}{n_{12} Z_2(s) + s M_{12}} \right] I_1(s) + n_{12} V_{d2}(s)
\]

(9)
$V_2(s) = n_{12}^2 M_{12} I_2(s) - \frac{1}{Z_2 + s \frac{M_{12}}{n_{12}}} V_2(s)$

where

$I_{d2}(s) = \frac{s M_{12}}{n_{12}(r_2 + s L_2)} I_1(s)$

$V_{d2}(s) = \frac{s M_{12}}{n_{12}} V_2(s)$

$Z_1(s) = r_1 + s(L_1 - n_{12} M_{12})$

and

$Z_2(s) = r_2 + s \left( L_2 - \frac{M_{12}}{n_{12}} \right)$

Derivations of (9) and (10) are given in Appendix B. Equation (9) describes the primary (input)-side circuit with a voltage-dependent voltage source $n_{12} v_{d2}$ in series with $Z_1$ and a parallel impedance of $\frac{s M_{12}}{n_{12}} Z_2$ and $s n_{12} M_{12}$. Equation (10) describes the secondary (output)-side circuit with a current-dependent current source $n_{12} i_{d2}$ in parallel with a series RL circuit, formed by the impedances $Z_2$ and $s \frac{M_{12}}{n_{12}}$. Fig. 9(b) shows the canonical transformer model.

### B. Circuit Implementation With Current Summing Technology

As mentioned in Section III-D, the connections between the parallel and the orthogonal windings should be interchanged, when the polarity of the lateral and angular misalignments are reversed. The circuit structure shown in Fig. 10(a) performs such function. The numerals “1,” “2,” and “3” appeared in the subscripts denote the transmitting coil, parallel winding, and orthogonal winding in the receiving coil, respectively. A capacitor $C_1$ is connected in series with the transmitting coil to resonate with the self-inductance $L_1$ of the coil [11], [31], so as to eliminate the effect of the series inductance and thus maximize the input power. Each output winding has a diode bridge connected. Fig. 10(b) illustrates how the canonical transformer model is applied. The outputs of the windings are current sources $n_{12} i_{d1}$ and $n_{13} i_{d3}$. Thus, in order to extract maximum currents from the windings, parallel resonant capacitors $C_2$ and $C_3$ are used. Such parallel resonance can also amplify the output voltages $v_{out2}$ and $v_{out3}$ to minimize the decrement of the efficiency caused by the forward voltage drop of the diode bridges. Since the phases of the output currents $i_{out2}$ and $i_{out3}$ are dependent on the ratio between the load impedance and the output parallel LC impedance, they can be of different values. Thus, if the outputs of the diode bridges are connected together, only a small phase difference between $i_{out2}$ and $i_{out3}$ can lead to circulating current and result in deenergizing one of the winding outputs. Thus, series inductors $L_{rect2}$ and $L_{rect3}$ with equivalent series resistances $r_{rect2}$ and $r_{rect3}$, respectively, are connected to the outputs of the diode bridges in order to energize both windings and deliver rectified and smooth output currents $I_{rect2}$ and $I_{rect3}$. Then, the outputs of the inductors are connected to the load $R_L$ together.

### C. Operating Modes of the Proposed Structure

As the mutual inductances $M_{12}$ and $M_{13}$ vary with the misalignment condition, there are three possible modes of operation, in which both windings or only one of the windings feed electric energy to the load. The equivalent circuits of the three modes are shown in Fig. 10(b)–(d). The operations are discussed as follows.

1) **Mode 1—Both windings transferring energy to the load:** In this mode, both windings process the energy transferred to the load. Fig. 10(b) shows the equivalent circuit model. The forward voltage drop of the diodes in the parallel and orthogonal windings is $V_{D2}$ and $V_{D3}$, respectively. The output voltages of the windings, $v_{out2}$ and $v_{out3}$, can be expressed as

$v_{out3} \approx \frac{H_3 v_{out2} - H_1 v_{in} - 2(V_{D2} - V_{D3})}{H_3}$

where $C_1 = \frac{1}{\omega L_1}, C_2 = \frac{1}{\omega^2 L_2}, C_3 = \frac{1}{\omega^3 L_3}$, and $\omega$ is the angular operating frequency.

$v_{out2} \approx \frac{H_4 + \frac{H_{r3}}{H_3} (Y_{r3} + K_3 H_4)}{\frac{1}{\pi R_L} + \frac{Y_{r2}}{r_{rect2}} \left( 1 + \frac{r_{rect2}}{R_L} \right)} + \frac{s V_{D2} + 2(Y_{r2} + K_2 H_4) (V_{D2} - V_{D3})}{\frac{1}{\pi R_L} + \left( K_2 H_4 + K_3 H_3 \right) H_3}$

Fig. 9. Equivalent circuit of a transformer. (a) Classical model. (b) Canonical model.
Derivations of (11) as shown bottom of the previous page and (12), and definition of the parameters are given in Appendix C.

If the dc resistances of the series inductors, \( r_{rect2} \) and \( r_{rect3} \), are much smaller than the load resistance \( R_L \), and \( V_{D2} \) and \( V_{D3} \) are assumed to be the same, \( r_{rect2} = r_{rect3} = 0 \) and \( V_{D2} = V_{D3} = V_D \). Equations (11) and (12) can be simplified as

\[
\begin{align*}
V_{out2} & \approx \frac{K_2 + K_3}{K_1} v_{in} + \frac{8}{\pi R_L} V_D + \frac{(K_2 + K_3)^2}{K_1} \frac{v_{out3}}{K_2} \\
V_{out3} & = v_{out2} = v_{out}.
\end{align*}
\]

The average load voltage \( V_L \) and the input current \( i_{in} \) are

\[
\begin{align*}
V_L & = \frac{2}{\pi} V_{out.pk} - 2V_D \\
i_{in} & = v_{in} - \frac{(K_2 + K_3) v_{out2}}{K_1}
\end{align*}
\]

where \( V_{out.pk} \) is the amplitude of \( v_{out} \).

2) Mode 2—Only parallel winding transferring energy to the load: This mode occurs when the transmitting coil and the parallel winding are near to the aligned condition. Fig. 10(c) shows the equivalent circuit model. \( v_{out2} \) and \( v_{out3} \) can be expressed as

\[
\begin{align*}
v_{out2} & \approx \frac{K_2 Y_{r3}}{R_L + r_{rect2} + \frac{8}{\pi (R_L + r_{rect2})} V_D} + \frac{\frac{8}{\pi (R_L + r_{rect2})} + \frac{K_2 Y_{r3}}{K_1 Y_{r3} + K_2 Y_{r2}}}{K_1 Y_{r3} + K_2 Y_{r2}} V_{D3} \\
v_{out3} & = v_{out2} = v_{out}
\end{align*}
\]

Derivations of (17) and (18), and definition of the parameters are given in Appendixes C and D

\[
\begin{align*}
V_L & = \frac{R_L}{R_L + r_{rect3}} \left( \frac{2}{\pi} V_{out2.pk} - 2V_D \right) \\
i_{in} & = \frac{v_{in} - (K_2 + K_3) v_{out2}}{K_1}
\end{align*}
\]

where \( V_{out2.pk} \) is the amplitude of \( v_{out2} \).

3) Mode 3—Only orthogonal winding transferring energy to the load: This mode occurs when the transmitting coil and the parallel winding are significantly misaligned. Fig. 10(d) shows the equivalent circuit model. \( v_{out2}, v_{out3}, \) and \( V_L \) can be expressed as

\[
\begin{align*}
v_{out2} & = \frac{K_2 v_{in} - K_2 K_3 v_{out3}}{K_1 Y_{r2} + K_2^2} \\
v_{out3} & \approx \frac{K_3 Y_{r3}}{R_L + r_{rect3}} + \frac{8}{\pi (R_L + r_{rect3})} + \frac{K_2 Y_{r2}}{K_1 Y_{r3} + K_2 Y_{r2}} + \frac{8}{\pi (R_L + r_{rect3})} V_{D3} \\
V_L & = \frac{R_L}{R_L + r_{rect3}} \left( \frac{2}{\pi} V_{out3.pk} - 2V_D \right)
\end{align*}
\]

where \( V_{out3.pk} \) is the amplitude of \( v_{out3} \).

The input current \( i_{in} \) in this mode has the same expression as (20).
D. Boundary Conditions for Determining the Operating Modes

The operating mode of the proposed circuit is determined by the ratio between $k_{12}$ and $k_{13}$, where $k_{12}$ is the coupling coefficient between the transmitting coil and the parallel winding, and $k_{13}$ is the coupling coefficient between the transmitting coil and the orthogonal winding.

In the Mode 1 operation

$$i_{\text{out}2} > 0 \quad \text{and} \quad i_{\text{out}3} > 0.$$  \hspace{1cm} (24)

In the Mode 2 operation

$$i_{\text{out}2} > 0 \quad \text{and} \quad i_{\text{out}3} = 0.$$  \hspace{1cm} (25)

In the Mode 3 operation

$$i_{\text{out}2} = 0 \quad \text{and} \quad i_{\text{out}3} > 0.$$  \hspace{1cm} (26)

The boundary between Modes 1 and 2 is determined by the conditions

$$v_{\text{out}2} = v_{\text{out}3}$$  \hspace{1cm} (27)

and

$$i_{\text{out}3} = 0.$$  \hspace{1cm} (28)

Define

$$\Psi = \frac{k_{12}}{k_{13}}.$$  \hspace{1cm} (29)

Based on (17) and (18), the circuit will be in Mode 2 operation if

$$\Psi \geq (\Psi_{M1-M2})^{-1}$$  \hspace{1cm} (30)

where $\Psi_{M1-M2} = \frac{1}{Q_3 \left[ \frac{1}{Q_2} - \frac{1}{Q_3} \left( 1 + Q_2 \right) \right]} \sqrt{\frac{L_2}{L_3}}$.

The expression and derivation of $\Psi_{M1-M2}$ are given in Appendix E.

Similarly, the boundary between Modes 1 and 3 is determined by the conditions

$$v_{\text{out}2} = v_{\text{out}3}$$  \hspace{1cm} (31)

and

$$i_{\text{out}2} = 0.$$  \hspace{1cm} (32)

Based on (21) and (22), the circuit will be in Mode 3 operation if

$$\Psi \leq \Psi_{M1-M3}$$  \hspace{1cm} (33)

where $\Psi_{M1-M3} = \frac{1}{Q_2 \left[ \frac{1}{Q_2} - \frac{1}{Q_3} \left( 1 + Q_3 \right) \right]} \sqrt{\frac{L_3}{L_2}}$.

Derivation of $\Psi_{M1-M3}$ is similar to the derivation of $\Psi_{M1-M2}$ in Appendix E, under the condition that $v_{\text{out}2} = v_{\text{out}3}$ and $i_{\text{out}2} = 0$.

Fig. 11 shows the boundaries for the three modes on the $k_{12} - k_{13}$ plane with three different turns-ratios between the parallel and orthogonal windings. They are 12:6, 12:12, and 12:24. Thus, both windings are concurrently energized within a wide range of coupling coefficient (i.e., in Mode 1 operation). The regions covered by Modes 2 and 3 is determined by the values of $L_2$ and $L_3$. When $L_3 > L_2$, the area covered by Mode 3 is more than that covered by Mode 2, and vice versa. For example, as shown in Fig. 11(a), $N_2 : N_3 = 12 : 6$, the area covered by Mode 2 is larger than the area covered by Mode 3. Conversely, as shown in Fig. 11(c), $N_2 : N_3 = 12 : 24$, the area covered by Mode 3 is larger than that by Mode 2.
V. POWER TRANSFER EFFICIENCY

The power transfer efficiencies of the parallel coils and the proposed structure are studied and compared. As shown in Fig. 1, the power transfer efficiency \( \eta \) is calculated by

\[
\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{V_L^2}{R_L} \text{Re}[v_{\text{in}}^*i_{\text{in}}^*]
\]

(34)

where \( P_{\text{out}} \) is the output power to the load \( R_L \), \( P_{\text{in}} \) is the input power to the transmitting coil, \( i_{\text{in}}^* \) is the conjugate of \( i_{\text{in}} \), and \( \text{Re}[v_{\text{in}}^*i_{\text{in}}^*] \) means the real part.

Based on the dimensions, positions, orientations, and number of turns of the coils, the mutual inductances \( M_{12} \) and \( M_{13} \) are obtained by using (3). The coupling coefficients \( k_{12} \) and \( k_{13} \) are determined by using the equations

\[
k_{12} = \frac{M_{12}}{\sqrt{L_1L_2}}
\]

(35)

and

\[
k_{13} = \frac{M_{13}}{\sqrt{L_1L_3}}
\]

(36)

The mode of operation is located by using the values of \( k_{12} \) and \( k_{13} \) on the \( k_{12} - k_{13} \) plane in Fig. 11. Then, the optimal load resistance \( R_{L,\text{optimal}} \) is determined by using an iterative method. Fig. 12 shows the flowchart. The steps are described as follows:

1) choose the magnitudes of the lateral misalignment \( \Delta \) and angular misalignment \( \theta \);
2) calculate \( M_{12} \) and \( M_{13} \) with (3), and determine the corresponding values of \( k_{12} \) and \( k_{13} \) with (35) and (36), respectively;
3) select an initial value of \( R_L \), e.g., 100 \( \Omega \), and set \( n = 1 \);
4) calculate \( \Psi \) with (29);
5) calculate \( \Psi_{M_{1-M2}} \) and \( \Psi_{M_{1-M3}} \) with (30) and (33), respectively;
6) determine the mode of operation, based on the criteria given in Section IV-D;
7) calculate the values of \( V_L \) and \( i_{\text{in}} \) with the required set of equations, (11)–(16) for Mode 1, (17)–(20) for Mode 2, and (20)–(23) for Mode 3;
8) calculate \( \eta(n) \) with (34) and compare it with \( \eta(n-1) \);
9) terminate the iteration with \( R_{L,\text{optimal}} = R_L(n) \) for the efficiency \( \eta \) at the considered misalignment if \( |\eta(n) - \eta(n-1)| < \epsilon_T \), and start from Step 1 with a new misalignment;
10) if not, increase the value \( R_L \) by \( \Delta R_L \), increment \( n \), and repeat from Step 5.

Fig. 13(a) shows the 3-D surface of the power transfer efficiency versus different combinations of \( \Delta \) and \( \theta \) of classical parallel coils. Fig. 13(b) shows the 2-D contour graph of Fig. 13(a). Fig. 13(c) and (d) shows the power transfer efficiency of the proposed coil structure with the windings having the turns ratio of \( N_2 : N_3 = 12 : 24 \). The physical dimensions and electrical parameters of the transmitting coil and receiving coil are given in Tables I and II. As shown in Fig. 13(a), the efficiency surface of the parallel coils is like a “rugby ball.” The rate of reduction of the efficiency versus lateral or angular misalignment becomes more severe when the efficiency is in the range of 30%, since the contour lines start getting closer from this range. The minimum efficiency is 0%, where the two coils are significantly misaligned. If the minimum efficiency considered is 30%, the maximum allowable lateral misalignment \( \Delta \) ranges from \(-30\% \) to \(+30\% \) and angular misalignment \( \theta \) ranges from \(-35^\circ \) to \(+35^\circ \). Such zone is shaded in Fig. 13(b).

The efficiency surface of the proposed structure shown in Fig. 13(c) is more uniform and no abrupt efficiency drop is observed. Again, if the minimum efficiency of 30% is needed, the maximum allowable lateral misalignment \( \Delta \) ranges from \(-45\% \) to \(+45\% \) and angular misalignment \( \theta \) ranges from \(-35^\circ \) to \(+35^\circ \). Such zone is shaded in Fig. 13(d). Thus, the misalignment tolerance of the proposed structure is better than the parallel coils.
VI. EXPERIMENTAL VERIFICATIONS

The performances of the parallel coils and the proposed structure are studied and compared on a testing setup shown in Fig. 14. Such setup allows altering and measuring the degree of lateral and angular misalignments. The parameters of the 16-turn transmitting coil are given in Table I. Three sets of receiving coils, including the classical parallel coil with 12 turns, proposed T-shaped structure with 12 turns in the parallel winding and six turns in the orthogonal winding, and another T-shaped structure with 12 turns in the parallel winding and 24 turns in the orthogonal winding, are fabricated. The physical dimensions and electrical parameters of the coils are given in Table II. They are measured by Agilent Impedance Analyzer 4294A. The components used in the summing circuit are given in Table III. The transmitting coil is driven by an RF amplifier, Amplifier Research 75A250A. The operating frequency is 2.2 MHz, which is typically used in retinal implant [32]. The value of the load resistance is determined by the procedure depicted in the flowchart shown in Fig. 12. It ensures maximum efficiency in the power transfer from the transmitting coil to the load.

The load voltage is observed by using a voltage probe (Model no. Tektronix P6139B) on the oscilloscope (Model no. Tektronix TDS3032). The input current is observed by using a current probe (Model no. Tektronix TCP202) on the same oscilloscope and the time delay (17 ns) for the current probe signal is taken into account. The input power is obtained by averaging the product of the input voltage $v_{in}$ and the input current $i_{in}$, where the time series of the voltage and current waveforms are first captured on the oscilloscope.

Fig. 6 shows the measured percentage variation of the mutual inductance between the transmitting and the parallel receiving coils under lateral and angular misalignment. Fig. 8 shows the measured percentage variation of $M_{12}$ and $M_{13}$ of the proposal coil with the turns ratio of $N_2 : N_3 = 12 : 24$ under lateral and angular misalignments.

With the parallel and orthogonal windings in the receiving coil connected as in Fig. 10(a), Fig. 15(a) shows the measured
Fig. 14. Experimental setup. (a) Aligned condition. (b) Lateral misalignment. (c) Angular misalignment.

A power loss audit of three cases in the three modes has been conducted. The results are shown in Table IV. It was found that the major loss is in the winding resistances. Thus, if the operating frequency is increased, the power loss in the windings will increase due to the skin effect and proximity effects [28].

Fig. 16 shows the waveforms of $i_{\text{out}2}$, $i_{\text{out}3}$, and $I_L$ under different combinations of lateral and angular misalignments with $i_{\text{out}2} : i_{\text{out}3} = 6 : 1$, $1 : 1$, and $1 : 6$. $I_L$ is found to be the sum of the rectified value of $i_{\text{out}2}$ and $i_{\text{out}3}$ in all cases, confirming the current summing technique. The waveforms of $i_{\text{out}2}$ and $i_{\text{out}3}$ are near square waveform, confirming the modeling of the corresponding current sources given in Section IV.

VII. OBSERVATIONS AND DISCUSSIONS

Based on the theoretical analysis and experimental results, some observations are made and future investigations are suggested.

1) Based on Figs. 13 and 15, it is confirmed that the orthogonal winding can effectively improve the transfer efficiency under coil misalignment. Thus, more orthogonal windings can be added to form complex loosely coupled winding structures. For example, a second orthogonal winding can be added onto the receiving coil. Then, the structure can tackle misalignment of a higher dimension. As the analysis method discussed in this paper starts from the basic principle, similar analysis can be adopted for studying the structure with the second orthogonal winding.

2) Compared Fig. 13(a) with Fig. 13(c), the overall transfer efficiency of the proposed structure will never go to zero within the considered range of the misalignments, because either one or both of the windings will have coupling with the transmitting coil. Conversely, the classical parallel coil could have zero efficiency.

3) The canonical transformer model presented in Section IV gives an alternative circuit model to study the coupling phenomenon of windings. Experimental results confirm the validity of the circuit model. Since the output winding is modeled as a current source in parallel with an inductance, it is suitable for modeling complex structures with several parallel windings.

4) The current summing technique with the added output inductors $L_{\text{rect}2}$ and $L_{\text{rect}3}$, shown in Fig. 10(a), can keep both windings energized to deliver power to the load. As shown in Fig. 11, except under extreme low values of the coupling coefficients $k_{12}$ and $k_{13}$, the entire circuit will operate in Mode 1, that is, both windings transfer energy to the load.

| TABLE III COMPONENT VALUES OF THE OUTPUT CURRENT SUMMING CIRCUIT |
|-----------------|-----------------|-----------------|
| $v_{\text{in, peak}}$ | $D_2 \sim D_3 \ast$ | $L_{\text{rect}2}$ and $L_{\text{rect}3}$ |
| 5 V | 0.92 mH | 15.0 $\Omega$ |

$\gamma = 1$ - 4.
TABLE IV
POWER LOSS AUDIT IN THE THREE MODES

<table>
<thead>
<tr>
<th>Mode</th>
<th>$P_{in}$ (mW)</th>
<th>$P_{out}$ (mW)</th>
<th>Loss of $r_1^\ast$ (%)</th>
<th>Loss of $r_2^\ast$ (%)</th>
<th>Loss of $r_3^\ast$ (%)</th>
<th>Loss of the diode bridges* (%)</th>
<th>Loss of $r_{rect2}$ and $r_{rect3}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>896</td>
<td>330</td>
<td>41.11</td>
<td>15.38</td>
<td>3.11</td>
<td>3.23</td>
<td>0.34</td>
</tr>
<tr>
<td>II</td>
<td>619</td>
<td>304</td>
<td>27.51</td>
<td>17.95</td>
<td>0.00</td>
<td>4.56</td>
<td>0.87</td>
</tr>
<tr>
<td>III</td>
<td>1448</td>
<td>173</td>
<td>75.46</td>
<td>2.31</td>
<td>9.61</td>
<td>0.66</td>
<td>0.02</td>
</tr>
</tbody>
</table>

*The losses are all normalized by the input power $P_{in}$.

5) As shown in Fig. 15, when the circuit is in Mode 2 operation, the transfer efficiency of the proposed structure is lower than with just the parallel coil. As shown in Fig. 10(c), it is mainly because a small amount of energy is dissipated in the orthogonal winding. In order to reduce the difference, the range of Mode 2 operation can be reduced by increasing the number of turns of the orthogonal winding. As shown in Fig. 15 (from green line to blue line), with the number of turns in the orthogonal winding increasing from 6 to 24, the range of the Mode 2 operation is reduced. As exemplified in Fig. 15, with the number of turns increased from 6 turns to 24 turns, the difference in the transfer efficiency is reduced from 9% to 1.7% in the lateral misalignment and from 10% to 1.7% in the angular misalignment. Such phenomenon can be explained by considering that the impedance of the orthogonal winding is increased. Thus, the current through the winding, and thus its power loss, are reduced.
Fig. 16. Waveforms of $i_{\text{out2}}$, $i_{\text{out3}}$, and $I_L$ under different misalignments
[Ch2: $i_{\text{out2}}$, Ch3: $i_{\text{out3}}$, Ch4: $I_L$ (50 mA/div)] (Timebase: 200 ns/div). (a) $i_{\text{out2}} : i_{\text{out3}} = 6 : 1$. (b) $i_{\text{out2}} : i_{\text{out3}} = 1 : 1$. (c) $i_{\text{out2}} : i_{\text{out3}} = 1 : 6$.

However, the number of turns cannot be continually increased, as there is a maximum achievable efficiency, similar to the characteristics of the parallel coil shown in Fig. 2. The difference of 1.7% is acceptable in practice, as there is a significant improvement in the efficiency under coil misalignment.

6) The efficiency drop issue discussed in item (5) may be solved by using a circuit to disable the orthogonal winding upon Mode 2 operation, such as the method discussed in [26]. However, special consideration should be given to the increase of the circuit complexity.

7) As shown in Fig. 15, when the circuit is in Mode 3 operation, the orthogonal winding takes up the main energy transfer. Since the coupling of the transmitting coil and the orthogonal winding is good, the transfer efficiency will increase abruptly.

8) It is understandable that the orthogonal winding can be placed in the transmitting coil and the receiving coil is a parallel winding. Driving of the transmitting coil can be similar to the one described in [8] and [24]. More complex winding structures can also be deduced.

9) Fig. 17 shows a model illustrating how the proposed structure can be placed inside an eyeball for a visual prosthesis [33], [34]. Electric power can be transmitted from the glasses. Complex structures can also be deduced for applications, like capsular endoscope, requiring dealing with high-dimension misalignment. Further investigations can be emphasized on the relationships among the operating frequency, coil size, and space constraint of the application.

VIII. CONCLUSION

A comprehensive investigation into the use of an orthogonal winding in loosely coupled link for enhancing transfer efficiency under coil misalignment has been presented. Both theoretical and experimental results confirm that the power transfer efficiency under coil misalignment can be effectively increased with the added orthogonal winding. A canonical transformer model has been derived and applied to study the proposed structure. Moreover, a current summing technique has been proposed to energize the parallel and orthogonal windings over a wide range of misalignments. The concept has been verified on a test bed.

APPENDIX A

EXPRESSIONS OF THE PARAMETERS IN (2)

\[ BD = R_1, BC = R_2, AC = R_3, AD = R_4, \cos \varepsilon = \frac{\alpha^2}{2lm}, \]
\[ \alpha^2 = R_4^2 - R_3^2 + R_2^2 - R_1^2, \]
\[ d^2 = R_3^2 - \mu^2 - \nu^2 + 2\mu\nu \cos \epsilon, \]
\[ \mu = \frac{l[2m^2(R_3^2 - R_3^2 - l^2) + \alpha^2(R_3^2 - R_3^2 - m^2)]}{4l^2m^2 - \alpha^4}, \]
\[ \nu = \frac{m[2l^2(R_3^2 - R_3^2 - m^2) + \alpha^2(R_3^2 - R_3^2 - l^2)]}{4l^2m^2 - \alpha^4}, \]
and
\[ \Omega = \tan^{-1} \left[ \frac{d^2 \cos \epsilon + (\mu + l)(\nu + m) \sin^2 \epsilon}{d R_1 \sin \epsilon} \right] - \tan^{-1} \left[ \frac{d^2 \cos \epsilon + (\mu + l) \nu \sin^2 \epsilon}{d R_2 \sin \epsilon} \right] + \tan^{-1} \left[ \frac{d^2 \cos \epsilon + \mu \nu \sin^2 \epsilon}{d R_3 \sin \epsilon} \right] - \tan^{-1} \left[ \frac{d^2 \cos \epsilon + (\nu + m) \sin^2 \epsilon}{d R_4 \sin \epsilon} \right]. \]

**APPENDIX B**

**DERIVATION OF (9) AND (10)**

Based on Fig. 9(a), the ideal transformer has the primary-side voltage \( v_{1T} \) and current \( i_{1T} \), and the secondary-side voltage \( v_{2T} \) and current \( i_{2T} \). Thus,
\[ V_{1T}(s) = n_{12} V_{2T}(s) \] \hfill (B1)
\[ V_{1T}(s) = s n_{12} M[I_1(s) - I_{1T}(s)] \] \hfill (B2)
\[ V_{2T}(s) = \left[ r_2 + s \left( L_2 - \frac{M}{n_{12}} \right) \right] I_{2T}(s) + V_2(s) \] \hfill (B3)
\[ sM[I_1(s) - I_{1T}(s)] = \left[ r_2 + s \left( L_2 - \frac{M}{n_{12}} \right) \right] I_{2T}(s) + V_2(s) \]
\[ I_{1T}(s) = \frac{I_{2T}(s)}{n_{12}} \]
\[ I_2(s) = -\frac{sM}{r_2 + sL_2} I_1(s) \]
\[ -\frac{1}{r_2 + s \left( L_2 - \frac{M}{n_{12}} \right) + s \frac{M}{n_{12}} V_2(s)} \]

By substituting (B6) into (B3) and (B1)
\[ V_{1T}(s) = \frac{s M}{n_{12}} \left[ r_2 + s \left( L_2 - \frac{M}{n_{12}} \right) \right] n_{12} I_1(s) \]
\[ + \frac{s M}{n_{12}} \left[ r_2 + s \left( L_2 - \frac{M}{n_{12}} \right) + s \frac{M}{n_{12}} \right] n_{12} V_2(s) \]
\[ V_1(s) = \left[ r_1 + s \left( L_1 - n_{12} M \right) \right] \]
\[ + \frac{sn_{12} M \left[ n_{12}^2 r_2 + sn_{12}^2 \left( L_2 - \frac{M}{n_{12}} \right) + sn_{12}^2 M \right]}{n_{12}^2 r_2 + sn_{12}^2 \left( L_2 - \frac{M}{n_{12}} \right) + sn_{12}^2 M} I_1(s) \] \hfill (B8)

\[ + \frac{s \frac{M}{n_{12}}}{r_2 + s \left( L_2 - \frac{M}{n_{12}} \right) + s \frac{M}{n_{12}}} n_{12} V_2(s). \] \hfill (B9)

**APPENDIX C**

**DERIVATIONS OF (11) AND (12)**

Let
\[ K_1 = r_1 + j \omega L_1 + \frac{1}{j \omega C_1} \]
\[ - \frac{(j \omega M_{12})^2}{r_2 + j \omega L_2} - \frac{(j \omega M_{13})^2}{r_3 + j \omega L_3} \]
\[ K_2 = \frac{j \omega M_{12}}{r_2 + j \omega L_2} \]
\[ K_3 = \frac{j \omega M_{13}}{r_3 + j \omega L_3} \]
\[ n_{12} v_{d2} = K_2 v_{out2}, \quad n_{13} v_{d3} = K_3 v_{out3} \] \hfill (C1)
\[ i_{in} = v_{in} - K_2 v_{out2} - K_3 v_{out3} \] \hfill (C2)
\[ n_{12} i_{d2} = K_2 i_{in}, n_{13} i_{d3} = K_3 i_{in}. \] \hfill (C3)

The admittances of the receiving parallel resonant circuits, \( Y_{rc2} \) and \( Y_{rc3} \), are
\[ Y_{rc2} = \frac{1 + j \omega C_2 (r_2 + j \omega L_2)}{r_2 + j \omega L_2} \]
\[ Y_{rc3} = \frac{1 + j \omega C_3 (r_3 + j \omega L_3)}{r_3 + j \omega L_3}. \]

When the inductance of \( L_{rect2} \) and \( L_{rect3} \) are large enough, \( L_{rect2} \) and \( L_{rect3} \) can be considered as constant dc current, so the current input to the rectifier \( v_{out2} \) and \( v_{out3} \) should be in form of square wave. The fundamental current components are
\[ i_{out2} = n_{12} i_{d2} - Y_{rc2} v_{out2}, \quad i_{out3} = n_{13} i_{d3} - Y_{rc3} v_{out3} \] \hfill (C4)
\[ |i_{out2}| = \frac{\pi}{4} v_{out2,\text{peak}}, \quad |i_{out3}| = \frac{\pi}{4} v_{out3,\text{peak}}. \] \hfill (C5)

By combining (C1)–(C5)
\[ I_{rect2} = |i_{out2}| = \frac{\pi}{4} \left[ \frac{K_2}{K_1} (v_{in} - K_2 v_{out2}) \right] \text{peak} \] \hfill (C6)
\[ I_{rect3} = |i_{out3}| = \frac{\pi}{4} \left[ \frac{K_3}{K_1} (v_{in} - K_2 v_{out2}) \right] \text{peak} \] \hfill (C7)
\[ V_{rect2} = \frac{2}{\pi} v_{out2,\text{peak}} - 2 V_D^2, \quad V_{rect3} = \frac{2}{\pi} v_{out3,\text{peak}} - 2 V_D^3 \] \hfill (C8)
\[ V_L = V_{rect2} - I_{rect2} v_{rect2}, \quad V_L = V_{rect3} - I_{rect3} v_{rect3}. \] \hfill (C9)
By combining (C6)–(C9)
\[
\frac{2}{\pi}v_{\text{out}2,\text{peak}} - 2V_{D2} - \frac{\pi}{4}i_{\text{out}2,\text{peak}}r_{\text{rect}2} = 2\frac{v_0}{\pi}v_{\text{out}3,\text{peak}} - 2V_{D3} - \frac{\pi}{4}i_{\text{out}3,\text{peak}}r_{\text{rect}3}\]  
(\text{C10})

When \(\omega L_{\text{rect2}}\) and \(\omega L_{\text{rect3}}\) are larger than the load resistance \(R_L\), \(v_{\text{out2}}\) and \(v_{\text{out3}}\) are in the same phase \(\varphi\)
\[
\frac{2}{\pi}v_{\text{out}2} - 2V_{D2}\angle\varphi = -\frac{\pi}{4}i_{\text{out}2}r_{\text{rect}2}\]
\[
= \frac{2}{\pi}v_{\text{out}3} - 2V_{D3}\angle\varphi = -\frac{\pi}{4}i_{\text{out}3}r_{\text{rect}3}\]

\[v_{\text{out}3} = H_2v_{\text{out}2} - H_1v_{\text{in}} - 2(V_{D2} - V_{D3})\angle\varphi]
(\text{H3})

where
\[
H_1 = \frac{\pi}{4}\left(\frac{K_2r_{\text{rect}2} - K_3r_{\text{rect}3}}{K_1}\right)
\]
\[
H_2 = \frac{2}{\pi} + \frac{\pi}{4}\left[\frac{K_2(K_2r_{\text{rect}2} - K_3r_{\text{rect}3}) + Y_{r2}r_{\text{rect}2}}{K_1}\right]
\]
\[
H_3 = \frac{2}{\pi} + \frac{\pi}{4}\left[\frac{K_3(K_3r_{\text{rect}3} - K_2r_{\text{rect}2}) + Y_{r3}r_{\text{rect}3}}{K_1}\right]
\]
\[V_L = \frac{\dot{V}_{\text{rect2}} - I_{\text{rect2}}r_{\text{rect2}}}{I_{\text{rect2}} = (I_{\text{rect2}} + I_{\text{rect3}})R_L}\]  
(\text{C12})

By substituting (C5) to (C8) into (C12), (C13) as shown at the bottom of the page, where \(H_4 = \frac{1}{\pi}\left[\frac{K_2}{K_1}\left(1 + \frac{r_{\text{out}2}}{r_{\text{rect}2}}\right) + K_3\right]\). Since \(C_1, C_2,\) and \(C_3\) resonate with the respective winding self-inductance, the phase angle of the output voltages \(\varphi\) is near zero.

**APPENDIX D**

**DERIVATIONS OF (17) AND (18)**

\[n_{13}i_{d3} = Y_{r3}v_{\text{out}3}\]
(\text{D1})
\[n_{13}i_{d3} = K_3\left(\frac{v_{\text{in}} - K_2v_{\text{out}2} - K_3v_{\text{out}3}}{K_1}\right).
(\text{D2})

By combining (D1) and (D2)
\[v_{\text{out}3} = \frac{K_1v_{\text{in}} - K_2K_3v_{\text{out}2}}{K_1Y_{r3} + K_2^2}I_{\text{rect2}} = \left|\dot{i}_{\text{out}2}\right| = \frac{\pi}{4}\left[\frac{K_2}{K_1}(v_{\text{in}} - K_2v_{\text{out}2} - K_3v_{\text{out}3})\right]
(\text{D3})
\]

\[\frac{2}{\pi}v_{\text{out}2,\text{peak}} - 2V_{D2} - \frac{\pi}{4}i_{\text{out}2,\text{peak}}r_{\text{rect}2} = \frac{\pi}{4}(\dot{i}_{\text{out}2,\text{peak}} + \dot{i}_{\text{out}3,\text{peak}})R_L
\]
\[\frac{2}{\pi}v_{\text{out}2} - 2V_{D2}\angle\varphi = -\frac{\pi}{4}i_{\text{out}2}r_{\text{rect}2}\]

\[v_{\text{out}2} = \left[H_4 + \frac{H_3}{H_5}(Y_{r3} + K_3H_3)\right]v_{\text{in}} + \frac{8}{\pi^2R_L}V_{D2}\angle\varphi + \frac{2(Y_{r3} + K_3H_3)H_3}{H_5}(V_{D2} - V_{D3})\angle\varphi
\]
(\text{C13})
\[
\frac{K_3}{K_2} = \frac{Y_{rc3}}{Y_{rc2} + \frac{s}{\pi^2 R_L}}
\]  
(E9)

By substituting \( K_2 \) and \( K_3 \) into (E9)
\[
\frac{M_{13}}{M_{12}} = \frac{1}{Q_3} \left( \frac{1}{Q_2} - \frac{s R_L}{\pi^2} (1 + Q_2) \right)
\]  
(E10)

where \( Q_2 = \frac{M_{12}}{L_1} \) and \( Q_3 = \frac{M_{13}}{L_3} \). \( M_{12} \) and \( M_{13} \) can be expressed in term of their corresponding coupling coefficient \( k_{12} \) and \( k_{13} \)
\[
M_{12} = k_{12} \sqrt{L_1 L_2} \quad \text{and} \quad M_{13} = k_{13} \sqrt{L_1 L_3}.
\]  
(E11)

By combining (E10) and (E11)
\[
\Psi M_{12} = \frac{k_{13}}{k_{12}} \left[ \frac{1}{Q_2} = f_{out2} = f_{out13}, f_{out1} = 0 \right]
\]  
\[
= \frac{M_{13}}{M_{12}} \sqrt{\frac{L_2}{L_3}} \left[ f_{out2} = f_{out3}, f_{out1} = 0 \right]
\]  
\[
= \frac{1}{Q_3} \left( \frac{1}{Q_2} - \frac{s R_L}{\pi^2} (1 + Q_2) \right) \sqrt{\frac{L_2}{L_3}}
\]  
(E12)

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