Nonlinear Coordinated Steering and Braking Control of Vision-Based Autonomous Vehicles in Emergency Obstacle Avoidance

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Abstract—This paper discusses dynamic control design for automated driving of vision-based autonomous vehicles, with a special focus on the coordinated steering and braking control in emergency obstacle avoidance. An autonomous vehicle is a complex multi-input and multi-output (MIMO) system, which possesses the features of parameter uncertainties and strong nonlinearities, and the coupled phenomena of longitudinal and lateral dynamics are evident in a combined cornering and braking maneuver. In this work, an effective coordinated control system for automated driving is proposed to deal with these coupled nonlinear features and reject the disturbances. First, a vision algorithm is constructed to detect the reference path and provide the local location information between vehicles and reference path in real time. Then, a novel coordinated steering and braking control strategy is proposed based on the nonlinear backstepping control theory and the adaptive fuzzy sliding-mode control technique, and the asymptotic convergence of the proposed coordinated control system is proven by the Lyapunov theory. Finally, experimental tests manifest that the proposed control strategy possesses favorable tracking performance and enhances the riding comfort and stability of autonomous vehicles.

Index Terms—Autonomous vehicles, nonlinear coordinated control, vision algorithm, fuzzy sliding mode control, steering and braking control.

I. INTRODUCTION

IN THE past two decades, such social concerns related to traffic accidents and energy consumption have increased rapidly [1]. Autonomous vehicles apply information, sense and control techniques to enhance driving safety and efficiency, which are regarded as one of the effective ways to improve traffic safety and reduce fuel consumption. Due to these potential benefits, recently, researches on autonomous vehicles have attracted more and more attentions.

The function of autonomous vehicles can be classified into two main aspects, known as assistant driving and automatic driving. Assistant driving, which is devoted to improve safety and riding comfort, is materialized by the emergence of new Advanced Driver Assistance Systems (ADAS), and the devices of ADAS, such as Adaptive Cruise Control (ACC), Forward Collision Avoidance (FCA) and Lane Departure Warning System (LDWS), are more and more available on the market. Automatic driving is the highest level of autonomous vehicles, and it is considered to be one of the toughest challenges in the exploitation of autonomous vehicles within the field of intelligent transportation system (ITS).

Automatic driving control system is a crucial component of autonomous vehicles in ITS, which mainly includes lateral and longitudinal motion control. The fundamental mission of lateral and longitudinal control is to automatically and accurately track the desired trajectory at the set speed while ensuring the safety, stability and riding comfort of autonomous vehicles [2].

A great deal of practice and research on the lateral motion control has been done in recent years. A nested PID steering control architecture with two independent control loops in vision-based autonomous vehicles is proposed and it can reject the disturbances on the curvature which increase linearly with respect to time [3]. In order to simulate human decision making and analogical reasoning, an intelligent fuzzy steering control strategy is given in [4] and [5]. Furthermore, an optimal fuzzy control system is constructed, in which the parameters of membership functions and rule base are determined by genetic algorithms [6]. In [7], a nonholonomic single-track vehicle model in local coordinates is given and a linear steering control law which can real-time reduce the tracking errors and avoid unpredictable overshoots is designed. In [8], a real world application of the lane-guidance technologies is discussed, and a new low-speed vehicle model that explains the source of the oscillation is proposed, the corresponding low-order steering controller is validated and refined through the LMI optimization synthesis. The input-output feedback linearization method is applied to the design of automatic steering control system in [9] and [10], however, the accurate knowledge of the plant dynamics needs to be known in advance. An active front steering system is designed by model predictive control (MPC) theory, and by introducing a constraint on the tire slip angle which stabilizes the vehicle at high speed, the performances of the proposed system is enhanced [11]. In [12], an adaptive fuzzy sliding mode lateral controller is proposed to deal with
parametric uncertainties and strong nonlinearities, and the asymptotic stability of the closed-loop lateral control system is proven. Moreover, Mammar et al. [13] design the assistant steering control system using hybrid automata theory and synthetically composite Lyapunov theory, and the practical implementation confirms the effectiveness of proposed approach.

The task of longitudinal control for autonomous vehicles is to track the desired velocity or the desired safe distance in real-time while maintaining stability and riding comfort. In [14], a nonlinear cascade longitudinal control system with inner and outer loops is proposed to ensure safety and comfort of autonomous vehicles. In the practical implementation of longitudinal velocity control, sliding mode control technique is a popular method [15]–[17], but, it is liable to cause the chattering phenomenon. An intelligent longitudinal vehicle following control system is developed in [18], and in this control system, the adaptive output recurrent cerebellar model articulation control (ORCMAC) is the main tracking controller to mimic an ideal backstepping control, and the robust controller is utilized to attenuate the effects caused by lumped uncertainty term. A vehicle spacing control system using robust fuzzy control with pole placement in an LMI region for TS model is described in [19], and the results indicate that the designed control law is robust enough to reject parametric uncertainties and the variations of operating conditions (e.g., wind, road surface). A longitudinal assistance control system including adaptive cruise control and forward collision warning/avoidance is developed in [20], which is adaptive to driver behavior, and the parameters of this presented control system is identified from the data in the manual operation phase. A novel time-varying parameter adaptive speed control algorithm is presented to improve the tracking capability under different working conditions, and the performance of the proposed control algorithm is validated by experimental tests [21].

Under the conditions of emergency obstacle avoidance, vehicle lateral and longitudinal dynamics has the strong coupled and nonlinear characteristics, and the coupled effects mainly embody in tire forces coupling, load-transfer coupling and kinematic coupling. In addition, the coupled effects become increasingly significant as maneuvers involving higher accelerations, larger tire forces, or reduced road friction. The performance of lateral and longitudinal controllers would be degraded if the features of coupling and nonlinearities of vehicles are neglected. Consequently, how to effectively and reasonably deal with the coupled behaviors between vehicle lateral and longitudinal dynamics is the emphasis and difficulty of motion control system design for autonomous vehicles [2].

In this paper, to deal with the coupled and nonlinear features of autonomous vehicles under the conditions of emergency obstacle avoidance, an coordinated steering and braking control system for automated driving is proposed. Firstly, a vision algorithm is constructed to detect the reference path and provide the local location information in real-time. Then, an adaptive nonlinear coordinated control strategy is proposed to overcome the strong nonlinearities and parametric uncertainties, and the asymptotic convergence of the proposed coordinated control system is proven by the Lyapunov theory. Finally, experimental tests manifest that the proposed control system possesses favor-
autonomous vehicles. The execution layer consists of executing devices and fault-tolerant system which has the actuator fault-diagnosis and fault-tolerant capabilities. As a consequence, the proposed automated driving control system has the features as follows [23].

1) Structure sharing for sophisticated and redundant system is adopted to improve source effectiveness and lower total cost.
2) Data information about traffic environment and vehicle status collecting from multiple sensors is fused.
3) Multi-objective coordinated control system for the lateral and longitudinal motions of autonomous vehicles is achieved to improve the system performance.

B. Vehicle Dynamics Model

Autonomous vehicle is a nonlinear multivariate system in the presence of strong coupled and uncertain properties. Since this paper focuses on studying the coordinated steering and braking control strategy, the driving input is not considered. The model is derived under the following assumptions: i) ignore vertical, roll, and pitch motion; ii) approximate the braking and steering dynamics as linear first-order systems; iii) discount the effect of suspension on the tire axles [2]. A simplified nonlinear vehicle dynamics model with three degrees of freedom (see Fig. 2) which can be effectively described in terms of longitudinal velocity, lateral velocity and yaw rate is

\[
\begin{align*}
\dot{v}_x &= -f_{Rg} - \frac{c_x v_y^2}{m} + v_y \dot{\psi} + 2C_f \frac{v_y + l_f \dot{\psi}}{mv_x} \delta_f + \frac{K_b P_b}{m} \tau_{br} \\
&\quad + g \sin \theta + \tau(\Delta_x) \\
\dot{v}_y &= -\frac{2(C_f + C_r)}{mv_x} v_y - \left[ v_x + \frac{2(C_f l_f - C_r l_r)}{mv_x} \right] \dot{\psi} \\
&\quad + 2C_f l_f \delta_f - \frac{c_y v_x^2}{m} + \tau(\Delta_y) \\
\dot{\psi} &= -\frac{2}{I_z} \left[ C_f l_f^2 + C_r l_r^2 \right] \dot{\psi} - \frac{2(C_f l_f - C_r l_r) v_y}{I_z v_x} \\
&\quad + \frac{2C_f l_f}{I_z} \delta_f + \tau(\Delta_\psi)
\end{align*}
\]

where \( v_x, v_y, \) and \( \psi \) represent the longitudinal velocity, the lateral velocity and the yaw angle, respectively. \( m \) is the total mass of vehicle. \( I_z \) is the yaw inertia. \( l_f \) and \( l_r \) are the distances of the front and rear axles from the CG, respectively. \( c_x \) and \( c_y \) are the longitudinal and lateral air resistance coefficients, respectively. \( f_{Rg} \) is the rolling resistance coefficient. \( C_f \) and \( C_r \) are the cornering stiffness of the front and rear tires, respectively. \( r_w \) is the vehicle radius, \( \theta \) is the road grade, \( \delta_f \) is the front wheel steering angle. \( P_b \) denotes the braking pressure, \( K_b \) denotes the braking pressure coefficient. \( \tau(\Delta_x), \tau(\Delta_y), \) and \( \tau(\Delta_\psi) \) denote the external disturbances and uncertainties caused by the time varying parameters and unmodeled dynamics. \( F_x, F_y, \) and \( M_z \) denote the total forces and moment acting on vehicle. \( \alpha_f \) and \( \alpha_r \) represent the tire slip angles. \( F_{xj}(j=f,r) \) and \( F_{yj}(j=f,r) \) represent the longitudinal and lateral tire forces, respectively.

The simplified vehicle dynamics model (1) can be rewritten in canonical form

\[
\begin{align*}
\dot{v}_x &= f_0 + g_0 \delta_f + g_1 P_b + \tau(\Delta_x) \\
\dot{v}_y &= f_1 + g_2 \delta_f + \tau(\Delta_y) \\
\dot{\psi} &= f_2 + g_3 \delta_f + \tau(\Delta_\psi)
\end{align*}
\]

with

\[
\begin{align*}
f_0 &= -f_{Rg} - \frac{c_x v_y^2}{m} + v_y \psi' \\
f_1 &= -2(C_f + C_r)v_y - \left[ v_x + \frac{2(C_f l_f - C_r l_r)}{mv_x} \right] \psi - \frac{c_y v_x^2}{m} \\
f_2 &= -2 \left( C_f l_f^2 + C_r l_r^2 \right) \psi - \frac{2(C_f l_f - C_r l_r) v_y}{I_z v_x} \\
g_0 &= \frac{2C_f(v_y + l_f \dot{\psi})}{mv_x}; \quad g_1 = \frac{K_b}{m r_w} \\
g_2 &= \frac{2C_f}{m} + g_3 = \frac{2C_f l_f}{I_z}.
\end{align*}
\]

Assumption 1: The uncertainties and external disturbances in the vehicle dynamics model (2) are limited in a certain range, and there exists known continuous functions \( \tau_i(i=1,2,3) \) which satisfy the following inequality conditions

\[
\begin{align*}
\tau(\Delta_x) &\leq \tau_1(v_x, v_y, \dot{\psi}) \\
\tau(\Delta_y) &\leq \tau_2(v_x, v_y, \dot{\psi}) \\
\tau(\Delta_\psi) &\leq \tau_3(v_x, v_y, \dot{\psi})
\end{align*}
\]

Steering and braking actuators are modeled as linear first order systems using the recursive least-square identification method, here, the transfer function models of steering and braking actuators are established as

\[
\begin{align*}
G_1(s) &= \frac{\delta_f}{\delta_{fd}} = \frac{M_1}{M_2 s + M_3} \\
G_2(s) &= \frac{P_{bd}}{P_{b,fd}} = \frac{N_1}{N_2 s + N_3}
\end{align*}
\]

where \( \delta_{fd} \) is the desired front wheel steering angle, \( P_{b,fd} \) is the desired braking pressure. \( M_1, M_2, M_3 \) and \( N_1, N_2, N_3 \) are the system parameters.

Due to the multiple driving requirements and dynamic cooperation of various components of autonomous vehicles, the technology of coordinated steering and braking control under the condition of emergency obstacle avoidance needs to be
studies. The task of longitudinal braking control is to guarantee the vehicles automatically and smoothly achieve the desired speed/acceleration by adjusting the braking pressure according to the specified control strategy. Given a desired velocity $v_d$, and an actual velocity $v_a$, the time derivative of velocity tracking error $e_v$ is defined as

$$
v_e = v_x - v_p
$$

The basic principle of steering control is to ensure the autonomous vehicles accurately track the planned reference trajectory, as shown in Fig. 3. The vision system can capture the real-time road scene and then determines the angular and lateral errors. In this paper, angular error $\phi_e$ is shaped by the vehicle centerline and the tangent of reference trajectory, lateral error $y_e$ is the horizontal distance between the vehicle position and the reference trajectory at a look-ahead distance $D_L$. The evolution of the measurements can be described as [6], [24]

$$
\phi_e = v_x K_L - \dot{\psi}
$$

$$
y_e = v_x \phi_e - v_y - \dot{\psi} D_L
$$

where $K_L$ is the road curvature.

### C. Vision Algorithm

Accurate and intact traffic environment information plays an important role that ensures the automatic driving control system of vehicles achieve the desired dynamic performance. With visual data that was grabbed from a single camera that is mounted on the roofline, the real-time vision system is mainly capable of estimating the vehicle location relative to the desired trajectory. Here, the proposed vision algorithm consists of five stages, the first few stages are responsible for trajectory detection, whereas the last stage realizes the curve fitting of the desired trajectory. The process of proposed vision algorithm is designed as follows.

**Image Filter and Enhancement:** Owing to the impact of surrounding background, images contain amount of noise during the process of generation and transmission. Firstly, the Gaussian filter method is adopted to reduce the influence of these noise disturbances. Then, a local contrast enhancement algorithm is adopted to effectively improve the whole or partial characteristics of image. For the point $(x, y)$ in the image, the implementation scheme of image enhancement is given as

1) Calculating the histogram equalization in a rectangular region with the center of point $(x, y)$ as

$$p_W(r_k) = \frac{n_k}{W^2}
$$

2) Establishing the cumulative distribution function $P_W(\cdot)$ as

$$P_W(r_k) = \sum_{i=0}^{k} p_W(i)
$$

3) Achieving the following gray transformation as

$$T(f(x, y)) = 255P_W(f(x, y))
$$

where $n_k$ is the number of pixels that have gray-scale $\theta_k$, $T(\cdot)$ is the gray transform function. The size of rectangular region $W$ is the only one control parameter in the local area histogram equalization.

**Edge Detection:** In order to obtain the edge information of shooting environment, Canny’s edge detection is proposed and the corresponding algorithm process is given as follows.

1) Using a Gaussian filter to eliminate the noise.
2) Employing the $3 \times 3$ Sobel operator to calculate the gradient values $(G_x, G_y)$ of the input image as shown in equations (12) and (13), the gradient magnitudes is calculated as equation (14), and the direction of the edges is determined as equation (15).

$$
G_x(x, y) = \{f(x+1, y-1)+2f(x+1, y)+f(x+1, y+1)
$$

$$
-\{f(x-1, y-1)+2f(x-1, y)+f(x-1, y+1)\}
$$

(12)

$$
G_y(x, y) = \{f(x-1, y+1)+2f(x, y+1)+f(x+1, y+1)\}
$$

$$
-\{f(x-1, y-1)+2f(x, y-1)+f(x+1, y-1)\}
$$

(13)

$$
G = \sqrt{G_x^2 + G_y^2}
$$

$$
\theta = \arctan\left(\frac{G_y}{G_x}\right)
$$

3) Applying the non-maximum suppression to suppress any pixel value that is not considered to be an edge.
4) Applying the double thresholding method to determine potential edges.

**Contour Extraction:** Edge images contain not only the information of the reference path, but also a number of little non-path. Usually, the reference path has the features of continuous and tenuous contour, however, most of the interference sources do not possess these features, as a consequence, the edge points of non-path can be removed based on the different profile features of path and non-path, here, a 8-neighbour contour extraction method is proposed to obtain the profile of each edge
chain [25]. After all the edges are found, the contour described by each edge chain is distinguished by the following principle.

1) Counting the number of edge points at each contour, the contour in which the number of edge points is below a set threshold is reviewed as non-path.
2) Calculating the envelop rectangle of the remaining contour, the contour that has the long and narrow rectangle is reviewed as non-path, and vice versa.

**Morphology Processing:** Due to the external disturbances, the edge information of reference path obtained by the above contour extraction is often intermittent, hence, mathematical morphology is applied to fill the gap among the interrupted edge points, here, the dilation and erosion operations are carried out for the same structuring element of edge images and given as

\[(f \circ b)(s, t) = \max \{f(s - x, t - y) + b(x, y)|s - x, t - y \in D_f, x + y \in D_b\}\]

\[(f \ast b)(s, t) = \min \{f(s + x, t + y) - b(x, y)|s + x, t + y \in D_f, x + y \in D_b\}. \quad (16)\]

The entity of the path can be highlighted by the dilation and erosion operations, and the location of path in images is not changed. The inner edge points of the left or right reference path can be detected from the center line to the both sides of processed images.

**Model Fitting:** In order to obtain the geometric model which can accurately describes the features of reference path, the model fitting is achieved by the least square method, assuming a simple data set \((x_i, y_i)\) consists of \(m\) points in the image, then the geometry model of reference path is usually in the form of a polynomial such as

\[y(x) = \sum_{j=0}^{m} c_j \varphi_j(x). \quad (17)\]

The goal of the problem is to seek for the values of \(c_0, c_1, \ldots, c_n\) such that the sum of square errors is minimized, it can be written as

\[s = \sum_{i=0}^{m} (y(x_i) - y_i)^2. \quad (18)\]

Fig. 4 indicates the extracted results of the proposed vision algorithm under different working conditions. It is interesting to note that the fitted curve model is in good coincidence with the actual reference trajectory under different illumination conditions. Meanwhile, the results manifest that the proposed vision algorithm provides a powerful guarantee to supply real-time location information between the vehicle and the reference trajectory for the follow-up coordinated steering and braking control system.

III. NONLINEAR COORDINATED STEERING AND BRAKING CONTROLLER

Under the conditions of emergency obstacle avoidance, the steering and braking dynamics of autonomous vehicles have the strong coupled, nonlinear and parametric uncertain features, and the performance of steering and braking motion controllers would be degraded if these characteristics are neglected. How to effectively and reasonably deal with the nonlinear and coupled behaviors between vehicular steering and braking dynamics is the emphasis and difficulty of automatic driving control system design for vehicles. In this section, As shown in Fig. 5, a coordinated steering and braking control system based on the nonlinear backstepping control theory and the adaptive fuzzy sliding mode control (FSMC) technique is constructed to guarantee uniformly ultimately bounded and global asymptotic stability of close loop system, and a major advantage of the proposed control strategy is that it has the greater flexibility to pursue the multi-objective control performances and effectively overcome the parametric uncertainties and nonlinearities [12].

A. **Nonlinear Backstepping Equivalent Control Strategy**

Nonlinear steering and braking coupled dynamics model of autonomous vehicles can be yielded by combining equations (2) and (7), (8), this vehicle dynamics model consists of six state variables and two input variables such as \(P_b\) and \(\delta_f\), which has semi-strict feedback form in the presence of external disturbances and parametric uncertainties. To deal with these features, a coordinated steering and braking equivalent control strategy based on nonlinear backstepping control technique is designed as follows.
Step 1: The first error vector $s_1$ is defined from the lateral error as

$$s_1 = y_e.$$  \hfill (19)

Choosing the Lyapunov function as $V_{lat0} = 1/2s_1^2$, and the time derivative of $V_{lat0}$ is obtained as

$$\dot{V}_{lat0} = s_1 \dot{s}_1 = s_1 \dot{y}_e = s_1(v_x \dot{\varphi}_e - v_y - \dot{\psi} D_L).$$ \hfill (20)

In equation (20), viewing the term $v_x \dot{\varphi}_e - \dot{\psi} D_L$ as the virtual control input, and the condition for which $\dot{y}_e$ tends towards zero is that $\dot{V}_{lat0}$ must be negative definite such that

$$\dot{V}_{lat0} = -k_1 s_1^2 \leq 0 \quad \text{where } k_1 \text{ is a positive constant.} \quad \text{Thus, the desired virtual control input } \alpha_1 \text{ can be obtained as}$$

$$\alpha_1 = -k_1 s_1 + v_y.$$ \hfill (22)

Defining the difference between the virtual control input $v_x \dot{\varphi}_e - \dot{\psi} D_L$ and its desired value $\alpha_1$ to be the second error variable $s_2$, and it is given by

$$s_2 = v_x \dot{\varphi}_e - \dot{\psi} D_L - \alpha_1.$$ \hfill (23)

Substituting equation (23) into equation (20), yields

$$\dot{V}_{lat0} = s_1 \dot{s}_2 - k_1 s_1^2.$$ \hfill (24)

Obviously, when $s_2 = 0$, $\dot{V}_{lat0} = -k_1 s_1^2 \leq 0$ is satisfied. The target of next step is to search the control input variables $P_b$ and $\delta_f$ which can ensure the error variable $s_2$ converge to zero or a small value. As a consequence, the error variable $s_1$ is guaranteed to asymptotically converge to zero or be uniformly ultimately bounded.

Step 2: Choosing the Lyapunov function as

$$V_{lat1} = V_{lat0} + \frac{1}{2} s_2^2.$$ \hfill (25)

The time derivative of equation (25) can be obtained as

$$\dot{V}_{lat1} = \dot{V}_{lat0} + s_2 \dot{s}_2 = -k_2 \dot{s}_1^2 + s_1 \dot{s}_2 + s_2 \dot{s}_2.$$ \hfill (26)

Let

$$-k_2 s_2 = s_1 + (f_0 + g_0 \delta_f + g_1 P_b) \varphi_e + \dot{\psi} \varphi v_x - (f_2 + g_2 \delta_f) D_L + k_1 \dot{s}_1 - (f_1 + g_2 \delta_f) + \eta_1.$$ \hfill (27)

Substituting equation (27) into equation (26), thus

$$\dot{V}_{lat1} = -k_2 \dot{s}_1^2 - k_2 s_2^2 \leq 0 \quad \text{where } k_2 \text{ is a positive constant, } \eta_1 \text{ is an uncertain term that caused by the time derivative of the error variable } s_2.$$ \hfill (28)

Based on the assumption 1, there exists known continuous positive function $\beta_1(v_x, v_y, \dot{\psi})$ which satisfies

$$\eta_1 \leq \beta_1(v_x, v_y, \dot{\psi}).$$ \hfill (29)

Step 3: Considering the longitudinal braking process of autonomous vehicles, the first error vector is defined as $p_1 = v_e$.

Choosing the Lyapunov function as

$$V_{log it0} = \frac{1}{2} p_1^2.$$ \hfill (31)

The time derivative of equation (31) is obtained as

$$\dot{V}_{log it0} = p_1 \dot{p}_1 = p_1 \dot{v}_e = p_1 (v_x - \dot{\psi}).$$ \hfill (32)

The condition for which $p_1$ tends toward zero is that $\dot{V}_{log it0}$ must be negative definite such that

$$\dot{V}_{log it0} = -l_1 p_1^2 \leq 0.$$ \hfill (33)

Let

$$f_0 + g_0 \delta_f + g_1 P_b + \xi_1 - \dot{\psi} = -l_1 p_1$$ \hfill (34)

where $l_1$ is a positive constant, $\xi_1$ is an uncertain term which caused by the time derivative of $p_1$.

Based on the assumption 1, there exists known continuous positive function $\gamma(v_x, v_y, \psi)$, which satisfies

$$\xi_1 \leq \gamma(v_x, v_y, \dot{\psi}).$$ \hfill (35)

Combining equation (27) and equation (34), the equivalent control input can be obtained as

$$u_{eq} = \begin{bmatrix} P_{bof} \\ \delta_{foe} \end{bmatrix} = \begin{bmatrix} g_1 \\ g_1 \varphi c + (g_0 \varphi_e - g_2 - g_3 D_L) \end{bmatrix}^{-1} \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}$$ \hfill (36)

with

$$\begin{bmatrix} \sigma_1 = -f_0 - \dot{\psi} - l_1 p_1 - \frac{g_0^2}{2 \xi_1} \\ \sigma_2 = -s_1 - f_0 \varphi_e - \varphi \varphi v_x + f_2 D_L + f_1 - k_1 \dot{s}_1 - k_2 s_2 - \frac{s_2^2}{2 \xi_1} \end{bmatrix}$$ \hfill (37)

where $-(p_1 \gamma^2/2 \xi_1)$ and $-(s_2 \beta_2^2/2 \xi_1)$ are nonlinear damping terms to compensate for the disturbances caused by the parametric uncertainties, and they exhibit lower gains at small tracking errors to enhance the riding comfort of vehicles and higher gains at large tracking errors to improve the safety of vehicles, both $\xi_1$ and $\xi_2$ are the positive constants.

Autonomous vehicles have the characteristics of parametric uncertainties and external disturbances. In order to restrain the influence of these uncertainties and disturbances, the variable structure reaching law is designed as follows:

$$u_s = \begin{bmatrix} P_{bs} \\ \delta_{fs} \end{bmatrix} = \begin{bmatrix} \lambda_1 \text{sign}(p_1) \\ \lambda_2 \text{sign}(s_2) \end{bmatrix}$$ \hfill (38)

where $\lambda_1$ and $\lambda_2$ are the positive constants, respectively, and $\text{sign}(\cdot)$ is the sign function.

**Remark 1:** The reaching law (38) is discontinuous across the sliding mode hyperplane, thus it will cause the high frequency chattering phenomena near the sliding hyperplane.

B. Adaptive Fuzzy Sliding Mode Reaching Law

In order to deal with the chattering problem caused by the sliding mode control law (38), an adaptive fuzzy sliding mode control scheme is proposed, and the two-input single-output
fuzzy logic control systems used for tuning reaching phase instead of sign function are constructed in the variable structure reaching control part. The main advantage of this method is that the robust behavior of the system is guaranteed. The second advantage of the proposed scheme is that the performance of the system in the sense of removing chattering is improving in comparison with the same sliding mode control technique without using fuzzy logic control [26]. The fuzzy variable structure reaching control law is rewritten as follows:

$$u_s = \frac{P_{bs}}{\delta_{fs}} = \left\{ \begin{array}{ll}
\lambda_1 u_{\text{FSMC}}(p_1, \dot{p}_1) \\
\lambda_2 u_{\text{FSMC}}(s_2, \dot{s}_2)
\end{array} \right. \quad (39)$$

In the fuzzy variable structure reaching control law (39), both of the fuzzy logic control systems taking the place of sign function are modeled with two input variables and one output variable. For the reaching control law of steering part, the two input variables are the sliding signal $s_2(t)$ and the rate of change of sliding signal $\dot{s}_2(t)$, respectively, two trapezoidal and five triangular membership functions are defined to depict each input variable, and seven single membership functions are defined to describe output variable. All membership functions are decomposed into seven fuzzy partitions expressed as positive small (PS), positive medium (PM) and positive big (PB), zero (ZE), negative big (NB), negative medium (NM), negative small (NS).

As seen in Table I, the rule base of fuzzy control system consists of 49 rules and represents as the mapping of the input and output linguistic variables, which can be defined heuristically in the following format:

$$R^{(i)} : \text{if } s_2(t) \in E_1^i \text{ and } \dot{s}_2(t) \in E_2^i \text{ then } u_{\text{FSMC}}(s_2, \dot{s}_2) = F^i$$

where $E_1^i$, $E_2^i$, and $F^i$ are the corresponding linguistic terms of the input and output fuzzy sets. $i = 1, 2, \ldots, 49$ is the number of the fuzzy if-then rule. For instance, a sample fuzz rule is given as

if $s_2(t)$ is negative big (NB) and $\dot{s}_2(t)$ is negative big (NB)

then $u_{\text{FSMC}}(s_2, \dot{s}_2)$ is positive big (PB)

It could be comprehended as the system states are below the sliding hyperplane and are moving away from it, therefore, in order to make the system states return to the sliding hyperplane quickly, the control action $u_{\text{FSMC}}(s_2, \dot{s}_2)$ should be PB.

The fuzzy inference is carried out by the Mamdani operator, and the implement of defuzzifier is utilized by the center of gravity method. For the regulation of braking dynamics part, the design flow of fuzzy logic system to take the place of sign function in reaching law (39) is same as above. Consequently, the total coordinated steering and braking control law can be expressed as

$$u = \left[ \begin{array}{c}
P_{bd} \\
\delta_{fs}
\end{array} \right] = u_{eq} + u_s. \quad (40)$$

**Theorem 1:** Consider the closed-loop system consisting of vehicle dynamics (2) and (7), (8) with the coordinated steering and braking controller (40), all signals in the closed-loop system are bounded, and the tracking errors asymptotically converge to zero.

**Proof:** With regard to the regulation of lateral dynamics, the following nonlinear steering control law can be obtained from the coordinated control law (40) as

$$u = \left[ \begin{array}{c}
P_{bd} \\
\delta_{fs}
\end{array} \right] = \left[ \begin{array}{c}
P_{beq} + \lambda_1 u_{\text{FSMC}}(p_1, \dot{p}_1) \\
\delta_{feq} + \lambda_2 u_{\text{FSMC}}(s_2, \dot{s}_2)
\end{array} \right]. \quad (41)$$

As seen in equation (36), it is interesting to note that the equivalent control terms of front steering angle $\delta_{feq}$ and braking pressure $P_{beq}$ satisfy the following equality as:

$$(f_0 + g_0 \delta_{feq} + g_1 P_{beq}) \varphi_e + \dot{\varphi}_e v_x - (f_2 + g_2 \delta_{feq}) D_L$$

$$+ k_1 \dot{s}_1 - (f_1 + g_2 \delta_{feq}) = -s_1 - k_2 s_2 - \frac{s_2 \beta^2}{2 \varepsilon_1}. \quad (42)$$

Defining Lyapunov function as

$$V_{\text{lat}} = V_{\text{lat1}} + \frac{1}{2} \dot{\varepsilon}_y^2 \quad (43)$$

where $\dot{\varepsilon}_y$ is the inevitable measurement error due to the lack of light and signal blockage of vision system, and it is assumed to be bounded as

$$|\dot{\varepsilon}_y| \leq \nu|s_2| \quad (44)$$

where $\dot{\varepsilon}_y$ is the time derivative of inevitable measurement error, and $\nu$ is the positive constants.

The time derivative of equation (23) is substituted into the equation (26), and then the following equality can be obtained as:

$$\dot{V}_{\text{lat}} = -k_1 \dot{s}_1^2 + s_1 s_2 + s_2 \dot{s}_2 + \dot{\varepsilon}_y \dot{\varepsilon}_y$$

$$= -k_1 \dot{s}_1^2 + s_1 s_2 + s_2 (\dot{\varphi}_e \varphi_e + \dot{\varphi}_e v_x - \ddot{\varphi} D_L - \dot{\alpha}_1) + \dot{\varepsilon}_y \dot{\varepsilon}_y$$

$$= -k_1 \dot{s}_1^2 + s_1 s_2 + s_2$$

$$\times (f_0 + g_0 \delta_{f} + g_1 P_{b}) \varphi_e + \dot{\varphi}_e v_x - (f_2 + g_2 \delta_{f}) D_L$$

$$+ k_1 \dot{s}_1 - (f_1 + g_2 \delta_{f}) + \eta_1 + \dot{\varepsilon}_y \dot{\varepsilon}_y. \quad (45)$$

Let $\delta_f = \delta_{fs}$ and $P_b = P_{bd}$, then, substituting the equivalent control law (36) and the fuzzy reaching law (39) into the above
equation (45), therefore, the equation (45) can be rewritten as
\[
\dot{V}_{lat} = -k_1 s_1^2 + s_1 s_2 + s_2 \dot{s}_2 + \tilde{e}_y \dot{e}_y \\
= -k_1 s_1^2 + s_1 s_2 + s_2 \left( -s_1 - k_2 s_2 - \frac{s_2 \beta^2}{2 \varepsilon_1} + \eta_1 \right) + \tilde{e}_y \dot{e}_y \\
+ s_2 \left( g_1 \varphi_c \delta_{bs} + (g_0 \dot{\varphi_c} - g_2 - g_3 D_L)\delta_{fs} \right) \\
= -k_1 s_1^2 - k_2 s_2^2 - \frac{s_2^2 \beta^2}{2 \varepsilon_1} + \eta_1 s_2 + s_2 \lambda_1 g_1 \varphi_c u_{FSMC}(p_1, \dot{p}_1) \\
+ s_2 \lambda_2 (g_0 \dot{\varphi_c} - g_2 - g_3 D_L) u_{FSMC}(s_2, \dot{s}_2) + \tilde{e}_y \dot{e}_y. 
\]  
\[ (46) \]
Since \( \eta_1 \leq \beta \), the polynomial term \(-s_2^2 \beta^2 / 2 \varepsilon_1\) + \( \eta_1 s_2 \) can be rewritten as
\[ \frac{s_2^2 \beta^2}{2 \varepsilon_1} + \eta_1 s_2 \leq - \left( \frac{s_2 \beta \sqrt{\varepsilon_1}}{\varepsilon_1} \sqrt{\varepsilon_1} \right)^2 + \frac{\varepsilon_1}{2} \leq \frac{\varepsilon_1}{2} \]  
\[ (47) \]
Assuming \( k_2 > k_1 \), let
\[ \kappa = \lambda_2 (g_0 \dot{\varphi_c} - g_2 - g_3 D_L) + \lambda_1 g_1 \varphi_c. \]
\[ (48) \]
The output of fuzzy logic system are normalized in the interval \((-1,1)\), then \( |u_{FSMC}(p_1, \dot{p}_1)| \leq 1 \) and \( |u_{FSMC}(s_2, \dot{s}_2)| \leq 1 \), and the equation (44) can be rewritten as
\[
\dot{V}_{lat} \leq -k_1 s_1^2 - k_1 s_2^2 - (k_2 - k_1) s_2^2 + \frac{\varepsilon_1}{2} + k|s_2| + \nu|s_2| \\
\leq -2k_1 V_{lat} - (k_2 - k_1) s_2^2 + \frac{\varepsilon_1}{2} + (k + \nu)|s_2| \\
\leq -2k_1 V_{lat} - \left( \sqrt{k_2 - k_1}s_2 \right)^2 + \frac{(k + \nu)^2}{2\sqrt{k_2 - k_1}} \\
+ \frac{\varepsilon_1}{2} - \frac{(k + \nu)^2}{4(k_2 - k_1)} \\
\leq -2k_1 V_{lat} + \frac{\varepsilon_1}{2}. 
\]  
\[ (49) \]
Consequently, \( \lim_{t \to \infty} V_{lat} \leq \frac{\varepsilon_1}{4k_1} \), the control error vector \( s_1 \) is uniformly ultimately bounded. Similarly, the error vector \( p_1 \) is uniformly ultimately bounded, proof is the same as above.

**Remark 2:** In order to effectively eliminate the chattering phenomenon and overcome the parametric uncertainties and external disturbances, a fuzzy logic system \( u_{FSMC}(p_1, \dot{p}_1) \) can be used to completely replace the sign function \( \text{sign}(p_1) \).

**IV. FIELD EXPERIMENTS AND DISCUSSIONS**

To confirm the performance of the coordinated steering and braking control system, both simulation and experimental tests which show the behaviors of the proposed control system are implemented, and the corresponding prototype vehicle is called Tiggo automated vehicle.

Firstly, the robustness of the coordinated control strategy against model uncertainties and disturbance is verified by simulation. The external disturbance is assumed as a random process, and the uncertain parameters of tire stiffness are changed from \( C_f = C_r = 50 \text{ KN} \) to \( C_f = C_r = 20 \text{ KN} \) in test.

The reference trajectory is straight, besides, the initial lateral and angular errors are set to 0.1 m and 0.04 rad, respectively. Fig. 6 shows the simulation results of proposed control system in the different working condition. It can be seen that the proposed system has strong robustness and high control accuracy with system uncertainties and disturbances.

Furthermore, experimental tests are carried out. To study the contribution of proposed controller, the dynamic behaviors of the proposed control system are analyzed and compared with the uncoordinated control system, which consists of a linear time varying steering controller [9] and a sliding mode longitudinal braking controller [15].

As shown in Fig. 7, the reference trajectory used in the experimental test I is consisted of several curve segments with different curvature radius, besides, Fig. 7 shows the desired velocity of autonomous vehicle in braking case, at first, the vehicle runs at a constant velocity of 90 km/h, then, it begins to decelerate since 40 m, in the final stage, it returns to run at a uniform velocity. Fig. 7 manifests that the longitudinal and lateral coupled and nonlinear dynamic features are occurred in experimental test I. The initial lateral, angular and velocity errors are set to 0.3 m, 0.05 rad, and \(-11 \text{ km/h}\), respectively.

A series of dynamic behaviors of the proposed coordinated control system and the LTV+SMC uncoordinated system are depicted in Fig. 8. Fig. 8(a) describes the response results of
lateral error, it can be seen that the maximum steady-state lateral error of the proposed control system and the LTV+SMC control system are bounded to ±0.1 m and ±0.2 m, respectively, which occurs in the tough road with largest curvature of 0.015 m⁻¹. Besides, the overshoot of lateral error controlled by the proposed control system is lower than the LTV+SMC control system. Fig. 8(b) shows the response results of angular error, it is clear that both the angular errors of proposed control system and LTV+SMC control system are limited and their maximum steady-state values are with in ±0.05 rad and ±0.1 rad, respectively. Fig. 8(a) and (b) manifest that the proposed control algorithm has less overshoot and smaller oscillation than the LTV+SMC control system.

Fig. 8(c) shows the response results of longitudinal velocity, it is interesting to note that the response curve of velocity for the coordinated control system basically coincided with the desired values. But, the response of longitudinal velocity for the LTV+SMC control system has a certain deviation, and the deviation is increased with the variations of path curvature.

Fig. 8(d) shows the comparison results of yaw rate, it can be observed that both the control strategies can ensure the yaw rate limit in a preconcert range, but the oscillation frequency of the LTV+SMC control system is enhanced obviously, which will make passengers uncomfortable. Consequently, compared with the LTV+SMC control strategy, the proposed control strategy can effectively decrease the oscillations and improve the control accuracy.

The contrasting results of corresponding front wheel steering angle and brake pressure are shown in Fig. 8(e) and (f), respectively. It is worth noting that the control inputs of the proposed coordinated control strategy are smoother than the LTV+SMC control strategy.

Fig. 9. Reference path in experimental test II.

The reference trajectory and velocity of autonomous vehicles used in the experimental test II are shown in Fig. 9. The initial lateral, angular and velocity errors are set to 0.4 m, −0.05 rad, and 4 km/h, respectively.

Fig. 10(a) and (b) describe the response results of lateral and angular errors, it can be seen that the tracking accuracy of the proposed control system is better than the LTV+SMC system. The maximum steady-state lateral error of the proposed control system is bounded to ±0.15 m, and the maximum steady-state angular error of the proposed control system is limited in ±0.05 rad.

Fig. 10(c) shows the response results of longitudinal velocity, it is worth noting that the proposed control system not only ensure the steady-state velocity error converge to zero, but also reject the adverse effects of the variations of path curvature. Nevertheless, the robustness of the LTV+SMC control strategy is relatively weaker.
Fig. 10(d) indicates that the proposed control strategy could effectively deal with the nonlinear features and take advantage of the interactions between the steering and braking dynamics to improve the riding comfort and stability of autonomous vehicles. The contrasting results of front wheel angle and brake pressure are shown in Fig. 10(e) and (f), respectively.

The comparative experimental results exhibited in this section manifest that the proposed coordinated control strategy not only significantly improves the control accuracy and yields transient performances, but also can enhance the riding comfort, stability and safety of autonomous vehicles.

V. CONCLUSION

This paper has presented a novel automated driving control system for the coordinated management of steering and braking dynamics of vision-based autonomous vehicles, which is aimed to effectively improve the safety and riding comfort properties.

The vision algorithm consisting of five stages is designed to real-time detect the desired path and provide the relative location information between the autonomous vehicle to the reference path.

Additionally, aiming at the coupled and nonlinear features of autonomous vehicles in the conditions of emergency obstacle avoidance, a nonlinear coordinated steering and braking control system consisting of a backstepping equivalent control law and a fuzzy sliding mode reaching control law is constructed, and the two-input single-output fuzzy logic control systems used for tuning reaching phase take the place of sign function in the reaching control law.

Furthermore, the overall proposed control system has been implemented on a prototype autonomous vehicle, and the results from the simulation and experimental tests demonstrate that the proposed control strategy possesses better tracking performances and enhances the riding comfort and stability of autonomous vehicles, even under adverse driving conditions.

REFERENCES


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