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Delay Analytical Models for Opportunistic Routing in Wireless Ad Hoc Networks

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***Abstract*—Opportunistic routing (OR) is a promising technology for improving the throughput of wireless networks. In contrast to traditional routing techniques that forward messages along predetermined paths, OR schemes allow selecting a set of candidate nodes as potential next-hop forwarders to utilize the broadcast nature of the wireless medium. An analysis of transmission latency and signaling overhead depend on the number of hops required to transmit a packet from source to destination. In this paper, we propose three models to analyze the number of transmissions for successfully delivering a packet with three OR schemes: single-forwarder OR with known topologies, multiple-forwarder OR with known topologies, and multiple-forwarder OR with unknown topologies. These models are useful in network planning. The simulation results verified the accuracy and usability of the three models.**

***Index Terms*—opportunistic routing, wireless ad hoc networks, delay analytical model.**

I. Introduction

Opportunistic routing (OR) is a promising technology for wireless networking. Most traditional multihop routing protocols send messages along a fixed path from source to destination. Because of the broadcast nature of the wireless medium, many nodes along the route that do not forward the packets still receive the message, which wastes precious network resources. To manage the error-prone nature of wireless media more effectively, OR was introduced [1]. This technology allows multiple nodes to participate in packet forwarding. One or more neighbor nodes are allowed concurrently to forward the packet to the next hop. OR has been shown to efficiently enhance network throughput [1]–[3] energy efficiency [4]–[7] and reliability [8]–[11] for wireless ad hoc networks.

One of the key problems in OR is the selection of forwarding nodes. The typical forwarder decision is composed of two processes: candidate selection and candidate coordination. Candidate selection involves determining a group of candidate forwarders and assigning scores to them based on certain criteria (e.g., expected transmission count (ETX) and expected any-path transmissions (EAX)). Then, these candidate forwarders exchange their scores, decide which forwarders will be the final next-hop forwarders, and complete the candidate coordination process. According to the number of forwarders, most OR transmission schemes can be classified into two categories: single-forwarder and multi-forwarder. Single-forwarder schemes have only one final forwarding node. ExOR [1] is a good example; in ExOR, each sender chooses its one-hop neighbors (i.e., those that directly receive the messages it sends) as candidate forwarders. All candidates that receive the packet must reply with an acknowledgement (ACK) message containing link information (e.g., ETX and EAX). With this, the candidate forwarder with the highest score is selected as the next-hop forwarder.

In multi-forwarder OR, all candidate forwarders that receive the packet can become next-hop forwarders. Because this approach may cause numerous duplicate transmissions, random network coding is usually adopted to optimize use of the received messages. MORE [2] is a state-of-the-art example. Upon receiving a packet, the node checks whether its identifier is on the packets forwarder list and whether the packet is innovative (i.e., linearly independent of the packets that have previously been received). If both conditions are satisfied, the packet is stored. The main aim of an OR protocol is to reduce the expected number of transmissions from source to destination while increasing the probability of reaching the destination. Over the year, numerous variants of OR protocols have been proposed for wireless networks. Some articles [12]–[18] have particularly emphasized opportunistic algorithms and protocols. In addition, many experiments and simulation studies have considered throughput, delay, and energy efficiency [19]–[21] in networks. All of these works are designed to improve candidate selection and coordination algorithms and to validate them through simulations or testbeds. Although several works have been proposed to study different aspects of OR, there are very little research focusing on analytical studies in OR. Among the few analytical studies in OR, they have mainly focused on the "single-forwarder" scenarios. For example, Lu and Wu [22] analyzed the compatibility of routing metrics to provide a guideline for protocol design. However, their model does not evaluate the transmission performance of routing protocols. In [23], a routing tree model was proposed to determine the average number of transmissions in OR routing procedures. Nevertheless, the scale of this model cannot extend easily because the complexity of closed-form formula increases exponentially as the network scale grows. Therefore, it is hardly usable when the number of nodes increases. [24] analyzes the lower bounds of energy-latency tradeoffs in multihop OR networks. The analysis is only applicable to the network organized in specific clustered structure, which makes it hard to be extended to model general OR protocols. [25] formulates the required time for successfully transmissions in different wireless routing protocols. Still, the analytical model provides a recursive formula that is difficult to calculate under

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high density. Also, their model is not suitable for analyzing multi-forwarder. In addition, their model requires complete knowledge of each link's packet delivery ratio; thus, extending it for use with networks of unknown topologies would be difficult. [26] gives an upper-bound of optimal performance in terms of delay (packet propagation speed) and shows the major delay is caused by packet retransmission. The impact of node density on end-to-end throughput/delay in wireless ad hoc networks was analyzed in [27]. These models can analyze the numbers of transmissions in OR but is only effective when the node density is known in single forwarder scenarios.

To the best of our knowledge, there is no work which provides a complete set of analytical results for wireless OR networks, including single-forwarder, multiple forwarder, known topology and unknown topology. However, there have been numerous publications in the analysis of routing in variety of wireless networks, including MANETs and delay-tolerant networks (DTNs). Existing works in DTNs have mainly focused on the delay performance. For example, the authors in [28] proposed a joint scheduling and drop policy to optimize the average delivery rate and delay. Furthermore, the work in [29] imposed lifetime constraints on packets and used a pure-birth CTMC model with the absorption state to evaluate the cumulative distribution function (CDF) and the nth order moment of delivery delay in Multicopy Two-Hop Relay Protocol. Similarly, [30] explored the impact of the number of selfish nodes and the intensity of their selfishness on information propagation. Matsuda and Takine [31] investigated delay-energy and delay-storage tradeoffs for (p, q)-routing algorithms. For analytical works in MANETs, they mainly studied two-hop relay algorithms. For example, Liu et al. [32] explored the performance of two-hop routing algorithm with erasure coding, proposing a general finite-state absorbing Markov chain, and studied the maximum throughput achievable in a MANET with interference from simultaneous transmissions and a general two-hop relay algorithm for packet routing [33]. In addition to upper bounds on delay, [34] also approximated end-to-end delay in MANETs. Regarding general MANETs with channel contentions, closed-form results on mean and variance of delivery delay were recently reported in [35]. Because there is no designated next-hop forwarder for each destination as in OR, the transmission behavior of DTNs/MANETs and OR is different. In addition, the nodes in DTNs and MANETs are mobile and utilize store-carry and forward transmission schemes; therefore, all of the above studies assume that the nodes are mobile and that nodes do not utilize overheard packets. For that reason, they cannot be directly applied to OR.

To summarize, there is no work that provides the analytical results for wireless networks from different aspects about performance that can be achieved by all kinds of OR, including single forwarder with known topology, single forwarder with unknown topology, multiple forwarders with known topology and multiple forwarders with unknown topology. The models we proposed are not discussed in the previous studies. In this paper, known topology means the information of all nodes position and channel quality in the network is known in a priori. On the other hand, unknown topology means the network in which all the information except for network density is not available for nodes in routing. In fact, from practical aspect, different analytical models are necessary in different circumstances because different network information (e.g., node positions, channel quality, network density, and number of forwarders) may be available in different scenarios.

Motivated by the above, this paper propose three analytical models for OR so that network managers can consider different aspects of the delay performance that can be achieved using OR. Given node placement/density and channel conditions, our models can estimate the number of transmissions between any two specific nodes. The assumptions of our models are very general and flexible, which does not require any mechanism for selection and prioritization of candidates; these models can be applied to OR networks with known topologies and to those with unknown topologies. As a result, the analysis of OR delay/numbers of transmissions could be used to help the following applications/aspects: For delay-sensitive applications (e.g., real-time service, video transmission, disaster relief, and military operations), the number of transmission must be feasible at or below a specific threshold. By using our models, the relationship between number of transmissions and node distance could be provided and the feasibility of transmission can be guaranteed. For network placement and planning, the nodes should be placed carefully for obtaining good performance. The nodes could be deployed in optimized positions to achieve required transmission delays when applying the proposed models. For power consumption in network, the transmission energy can be evaluated and reduced through using our models because the power consumption of source/destination pairs is directly related to the total number of transmissions. In short, from practical aspects, our models could provide guidelines for network management, node deployment, and network planning optimization for wireless OR networks.

In this paper, we propose analytical models for three OR scenarios: a single forwarder with known topology (SF/KT), multiple forwarders with known topology (MF/KT), and multiple forwarders with unknown topology (MF/UT). The accuracy levels of the proposed analytical models are verified using simulations. The remainder of the paper is organized as follows: In Section II, the system model is defined, and the analytical models are described. In Section III, the analytical results are validated using simulations, and additional properties are observed. Finally, the paper is concluded in Section IV.

II. Analysis of number of transmissions

In this section, we will study the performance behavior of SF and MF under KT and UT in terms of the number of transmissions. . Because the SF/UT scenario has been studied in [27], in this paper, we will focus on the remaining three combinations, i.e., SF/KT, MF/KT, and MF/UT. For the SF/KT scenario, we analyze the number of transmissions given the network topology (Theorem II.1). For MF/KT, because calculating the actual results of the recursive formula in Theorem II.2 would be difficult, Theorems II.3 and II.4 respectively

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TABLE I: Notation used in the analysis

Symbol	Description
$Pt_{i,j}$	The delivery probability of the edge between the pair of nodes i and j.
$Pd_{i,j}$	The probability that the dedicated next-hop relay node receives the packet sent by node i is j.
B_i	The neighboring set of node i.
$\omega(i)$	The approximation degree to d of node i.
F_i	The set of candidate forwarders of node i.
k	The maximum number of candidate forwarder
$Pt(r)$	The successful packet reception probability at distance r from the transmitter.
D	The distance from source to destination.
a	Transmission range of a node.
λ	Network density, i.e., the number of nodes in a unit area.
$P^*_{R_n}$	The probability that a node inside the sector R_n receives the packet send by s in a timeslot.
$P^*_{\tilde{R}_n}$	The probability that a node inside the sector $\tilde{R_n}^*$ receives the packet send by s in a timeslot.
P_{R_n}	The probability that at least one node in sector R_n receives the packet.
$P_{\tilde{R}_n}$	The probability that at least one node located farther than r_{n+1} receives the packet.
Y_λ	The random variable denoting the distance from the source to the next forwarding node with network density λ.

provide the upper bound and lower bound for the problem. For MF/UT, the expected hop progress is determined from the network density (Theorem II.5). Corollaries II.5.1 and II.5.2 provide the required numbers of transmissions and the relationship with network density in the MF/UT scenario.

A. System Model and Notation

We first consider a general multihop wireless network where the message is transmitted from source s to destination d. In the network, each node i's "approximate degree to d" is denoted by $\omega(i)$, which can be measured by the distance or hop count between i and d. Then, each node can be indexed with an integer in the increasing order of $\omega(i)$ (i.e., $\omega(1) < \omega(2) < ... < \omega(N)$), where N is the total number of nodes. The neighboring set of node i (i.e.,those within node i's transmission range) is denoted by B_i.The candidate set of node i (i.e., those that may be selected to be next-hop forwarders) is denoted by F_i. Nodes in F_i must satisfy the following three conditions: 1) they must be node i's neighbors so that they can receive the message (i.e.,$F_i \subseteq B_i$); 2) they must be closer to the destination than node i (i.e., $\forall j \in F_i, \omega(i) < \omega(j)$), and 3) the number of candidates is limited to a predetermined limit k (i.e.,$|F_i| < k$) because of coordination overhead limits.

To describe a certain node in F_i, let $F_i(j)$ denote the jth candidate in F_i (i.e.,$\omega(f_i(1)) < \omega(f_i(2)) < ... < \omega(f_i(|F_i|))$). Because the channel quality varies for each pair of nodes, we let $Pt_{i,j} > 0$ denote the probability of a successful packet delivery from node i to j in a single transmission. Depending on different OR designs, some candidate forwarders may have successfully received the packet but may not necessarily forward the packet. Therefore, we let $Pd_{i,j}$ denote the probability

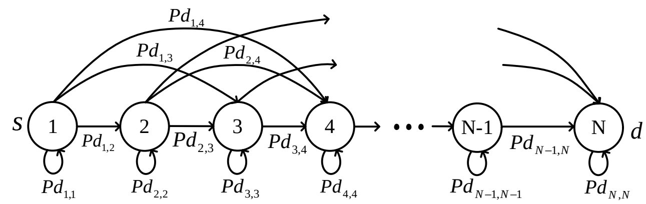

Fig. 1: Markov chain with single absorbing state

that "node j becomes a forwarder and transmits the received packet at the next time-slot." In this paper, our goal is to analyze $E[T_{s,d}]$, the expected number of timeslots to transmit a packet from s to d. We assume that all nodes in OR are loosely synchronized and can only transmit packets in the beginning of the timeslot.

Therefore, $T_{s,d}$ refers to the number of timeslots for a packet transmission from source to destination. The notation used in the analysis is summarized in Table I.

B. Single Forwarder with Known Topology

First, we analyze the numbers of transmissions for the "single forwarder OR with known topology (SF/KT)" scenario, which refers to OR schemes in which only one next-hop forwarder is allowed after each nodes transmission. Each forwarding node keeps transmitting until the packet has been successfully received by one of its neighbors. Among the receiving candidates, the one with the highest score will forward the packet to the next hop. Therefore, given $Pt_{i,j}$ of all pairs of nodes and $j \in B_i$, $Pd_{i,j}$ is

$$Pd_{i,j} = Pt_{i,j} \prod_{l=j+1}^{|F_i|} \left(1 - Pt_{i,f_i(l)}\right) \quad (1)$$

When node i transmits a packet, the receiving node becomes the next-hop forwarder only if 1) it receives the packet from node i and 2) no other nodes with higher scores receive the packet. Because there is only one transmitting node in the network at each timeslot, we can describe SF/KT with a Markov chain that has a single absorbing state (i.e., destination).

In the transformed diagram, each state represents the current receiving node; each transition between two states refers to a successful transmission between these two nodes; and the absorbing state refers to the destination. If the packet is successfully transmitted, it moves to the next state; otherwise, it stays in the current state. This is repeated until state N (i.e., destination) is reached. Therefore, the probabilities of staying in the current state can be expressed as:

$$\begin{cases} Pd_{i,i} = \prod_{h \in F_i} (1 - Pt_{i,h}) & \text{if } i \neq d \\ Pd_{i,i} = 1 & \text{otherwise} \end{cases} \quad (2)$$

The Markov chain and the corresponding state transition is depicted in Fig. 1. To find $E[T_{s,d}^{SF/KT}]$, we build an upper-triangular matrix with size $N \times N$ (i.e., $D_{s,d} \in [0,1]_{N \times N}$), which consists of the next-hop forwarder transition probability $Pd_{i,j}$ for all i and j.

$$D_{s,d} = \begin{bmatrix} Pd_{1,1} & Pd_{1,2} & \cdots & Pd_{1,N} \\ 0 & Pd_{2,2} & \cdots & Pd_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Pd_{N,N} \end{bmatrix}_{N\times N}$$

For convenience, $D_{s,d}$ can be expressed in its canonical form as follows:

$$D_{s,d} = \begin{bmatrix} Q & R \\ 0 & 1 \end{bmatrix}$$

where Q is the probability of transmissions before the destination is reached,$R = [Pd_{1,N} Pd_{2,N} ... Pd_{N-1,N}]^T$ represents the probabilities to reach the destination in one transmission from nodes $1, 2, \cdots, N-1$, and N is the absorbing state of the Markov chain. Using these matrices, we can find the expected numbers of transmissions for the SF/KT scenario with Theorem 1-1.

Theorem II.1. *In an N-node "single forwarder OR with known topology" network, the expected number of transmissions from source to destination is the first entry of vector **e** = F**1**, where $F = (I + Q + Q^2 + ...)$=$(I - Q)^{-1}$, I is the identical matrix and **1** is a length- $N-1$ column vector whose entries are all ones.*

Proof: To find the expected number of visits before absorption, we start from the kth power of $D_{s,d}$; $D^k_{s,d}$ shows the state transition probability after k steps, as follows:

$$D_{s,d} = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}, ...,$$

$$D^k_{s,d} = \begin{bmatrix} Q^k & (I + Q + Q^2 + ... + Q^{k-1})R \\ 0 & I \end{bmatrix}.$$

Note the probability of transition from node i to j at exactly k steps is the (i, j)-entry of Q^k.

$$D^{\infty}_{s,d} = \begin{bmatrix} Q^{\infty} & (I + Q + Q^2 + ...)R \\ 0 & I \end{bmatrix} = \begin{bmatrix} Q^{\infty} & FR \\ 0 & I \end{bmatrix}$$

Next, we sum up all k (from 0 to ∞) in the state matrix and obtain the fundamental matrix(denoted by F). It is straightforward to prove that $F = \sum_{x=0}^{\infty} Q^x = (I - Q)^{-1}$, where I is the 1×1 identity matrix.

The (i, j) entry of matrix F is the expected number of times the chain is in state j, given that the chain started in state i. Finally, the first entry of **e** is the expected number of transmissions from source to destination for single forwarder OR, where **e**=F**1**. ■

C. Multiple Forwarder with Known Topology

Next, we analyze the numbers of transmissions of MF/KT OR networks.In MF/KT, all candidate forwarders are allowed to forward the successfully received packet. Therefore, for all nodes j, $k \in F_i$, $Pd_{j,k} = Pt_{j,k}$.Because there are multiple receiving nodes at each timeslot and each node has an independent probability of receiving the packet, the state of the network is infeasibly large if we transform MF/KT into a Markov chain. Therefore, we use the expected hop distance (i.e., distance traversed by the message per timeslot) to evaluate the numbers of transmissions. As depicted in Fig. 2, we model the nodes as a linear topology, considering the distance to the destination d. Let $d_{i,j}$ be the distance between any two nodes i and j. Let H_n denote the random variable of the distance progressed after one transmission and $n = |F_s|$.

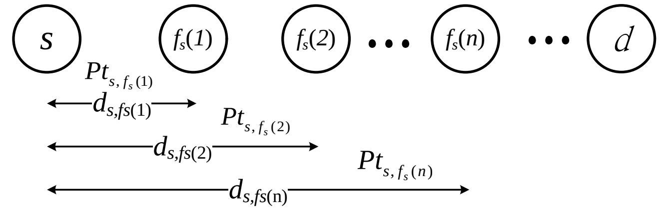

Fig. 2: Linear topology with distances to destinations

1) Expected Numbers of Transmissions of MF/KT:

Theorem II.2. *For the multiple forwarder OR with known topology, because the distance from source to destination is D,the expected number of transmissions is $E[T^{MF/KT}_{s,d}] = D/E[H_n]$, where $E[H_n] = d_{s,f_s(n)} Pt_{s,f_s(n)} + (1 - Pt_{s,f_s(n)})$ $E[H_{n-1}]$.*

Proof: By definition, random variable H_n denotes the distance progressed after one transmission with n candidate forwarders, and $Pr\left\{H_n = d_{s,f_s(j)}\right\}$ denotes the possibility that the packet traverses from s to $f_s(j)$; that is, $Pr\left\{H_n = d_{s,f_s(j)}\right\} = Pt(d_{s,f_s(j)}) \prod_{l=j+1}^{n} (1 - Pt_{s,f_s(l)})$.

The reason is that node $f_i(j)$ becomes the most distant forwarder only if 1) it receives the packet from node i and 2) no other nodes with higher scores receive the packet from node i. Then, the expected hop progress of MF/KT can be expressed as: $E[H_n] = \sum_{j=1}^{n} d_{s,f_s(j)} Pr\left\{H_n = d_{s,f_s(j)}\right\}$, which can be further calculated as:

$$\begin{aligned} E[H_n] &= d_{s,f_s(n)} Pt_{s,f_s(n)} + d_{s,f_s(n-1)} Pt_{s,f_s(n-1)} (1 - Pt_{s,f_s(n)}) \\ &\quad + ... + d_{s,f_s(1)} Pt_{s,f_s(1)} \prod_{j=2}^{n} (1 - Pt_{s,f_s(j)}) \\ &= d_{s,f_s(n)} Pt_{s,f_s(n)} + (1 - Pt_{s,f_s(n)}) E[H_{n-1}] \qquad (3) \end{aligned}$$

Based on $E[H_n]$, the expected number of transmissions is given by $E[T^{MF/KT}_{s,d}] = D/E[H_n]$ ■

The time required for calculating (3) has a complexity of $O(n!)$, where n is the number of candidate forwarders. Therefore, for rapid calculation, we evaluate the upper and lower bounds for the numbers of transmissions with MF/KT in Theorems II.3 and II.4.

2) Upper Bound of Number of Transmissions:

Theorem II.3. *For MF/KT, given the distance from source to destination D, the first entry of vector **e** $\geq E[T^{MF/KT}_{s,d}]$(i.e., the upper bound of MF/KTs number of transmissions is the first entry of vector **e** =F**1**),which is the expected number of transmissions from source to destination in SF/KT.*

Proof: The rationale behind this upper bound is intuitive. In MF/KT, all the neighbor nodes of the relay node can

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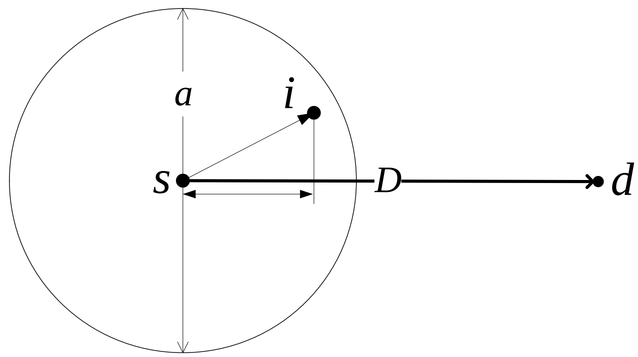

Fig. 3: Definition of forward progress

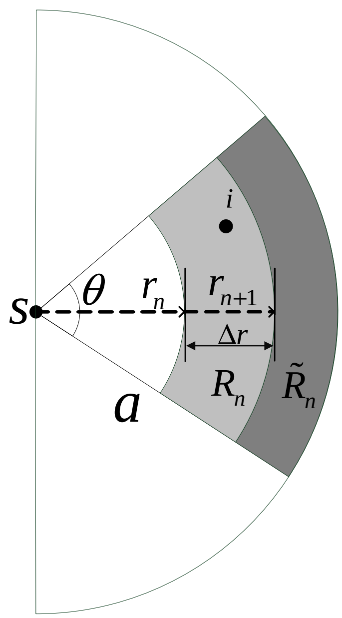

Fig. 4: Semicircle model for MF/UT

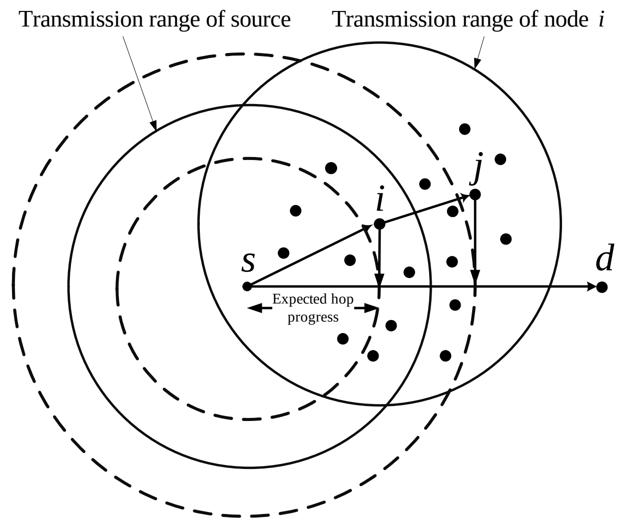

Fig. 5: Transmission in MF/UT

be candidate forwarders, the set of which has not fewer than one forwarder in SF/KT. Given the same transmission probability between two nodes, the hop progress of MF/KT is always larger than that of SF/KT. Therefore, the number of transmissions of SF/KT (i.e., first entry of vector **e**) is the upper bound of the number of transmissions of MF/KT. ■

3) Lower Bound of Number of Transmissions:

Theorem II.4. *For MF/KT,* $E[T_{s,d}^{MF/KT}] \geq 1/(1-\prod_{i=1}^{N-1}(1-Pt_{i,N}))$.

Proof: At the start of an MF/KT transmission, only the source node has packets to send, and it takes time for other nodes to obtain the packet and become forwarders. Therefore, the expected number of transmissions of the saturation condition (i.e., every node except the destination become forwarders) can serve as the lower bound of MF/KT. The successful transmission probability of the saturation condition can be expressed as $1 - \prod_{i=1}^{N-1}(1 - Pt_{i,N})$ because the destination receives the packet as long as at least one node transmits successfully. Therefore, the expected number of timeslots for such a condition is

$$\sum_{x=1}^{\infty} x(1 - \prod_{i=1}^{N-1}(1 - Pt_{i,N}))(\prod_{i=1}^{N-1}(1 - Pt_{i,N}))^{x-1}$$
$$= 1/(1 - \prod_{i=1}^{N-1}(1 - Pt_{i,N})).$$

■

From Sections II.C.2 and II.C.3, we know the upper and lower bounds of the number of transmissions. To evaluate the number of transmissions of MF/KT quickly, we suggest using the mean value of the upper and lower bounds.

D. Multiple Forwarder with Unknown Topology

The previous two models (i.e., SF/KT and MF/KT) require detailed information of the network topology (i.e., transmission probabilities for between each pair of nodes). Since it is not always easy to obtain accurate positions and channel conditions under a large and dense wireless networks, we propose an approximating approach to analyze the number of transmissions for MF/UT OR networks. With this model, one can easily estimate the expected delay between two nodes with a given distance between the source and destination and a certain network density. Note that the analysis of known topology and unknown topology are based on different information. Unlike known topology scenario, for unknown topology, the transmission probability is defined as "the expected successful rate for hopping a given distance under a certain node density". Because the position of each node is unknown, we designate $Pt(r)$ to represent the probability of a successful packet transmission between two nodes separated by a distance r. $Pt(r)$ can be obtained using existing channel models or field-test results. In this paper, we assume that the nodes are uniformly distributed on a 2D plane with a spatial Poisson distribution with average density λ. Each node has the same transmission range a.

1) Hop Progress: We aim to determine the probability density function (PDF) of hop progress under given network density. As shown in Fig. 3, we define hop progress as the one-hop distance between nodes s and i, as projected onto a line drawn from node s to the destination d. Let Y_λ be a random variable of the hop progress after a transmission with density λ, and let $f_\lambda(y)$ and $F_\lambda(y)$ denote the PDF and the CDF of Y_λ, respectively. The expected value of the hop progress (i.e., $E[Y_\lambda]$) can be obtained as follows: To analyze the hop progress, we use a semicircle leading toward the destination under each transmission, as depicted in Fig. 4. We divide this semicircle into many small arcs by radii $(r_1, r_2, ..., r_n, ...)$, for all i, $r_{i+1} - r_i = \Delta r$. Here, R_n represents the area encircled by the areas with radii r_n and r_{n+1}, whereas $\tilde{R}_n$ denotes the remaining area beyond r_{n+1}. We further let $P^*_{R_n}$ denote the probability that "a node inside the sector R_n receives the packet sent by s in a timeslot" and let $P^*_{\tilde{R}_n}$ denote the probability that "a node inside the sector $\tilde{R}_n$ receives the packet sent by s in a timeslot." With $P^*_{R_n}$ and $P^*_{\tilde{R}_n}$, we can obtain $P_{R_n}/P_{\tilde{R}_n}$, which represents the probability that "there exists at least one node receiving the packet in $R_n/\tilde{R}_n$." We first determine $P^*_{R_n}$, which can be described as the conditional probability that a node receives the packet when this node is inside region R_n with network density λ, as shown in Fig. 4. Hence, we have

$$P^*_{R_n} = \frac{\int_0^\theta \int_{r_n}^{r_{n+1}} Pt(r) r dr d\theta}{\int_0^\theta \int_{r_n}^{r_{n+1}} r dr d\theta} = \frac{\int_0^\theta \int_{r_n}^{r_{n+1}} Pt(r) r dr d\theta}{\frac{1}{2}(r_{n+1}^2 - r_n^2)\theta}. \quad (4)$$

Similarly, $P^*_{\tilde{R}_n}$ can be represented as

$$P^*_{\tilde{R}_n} = \frac{\int_0^\theta \int_{r_n+1}^{a} Pt(r) r dr d\theta}{\int_0^\theta \int_{r_n+1}^{a} r dr d\theta} = \frac{\int_0^\theta \int_{r_n+1}^{a} Pt(r) r dr d\theta}{\frac{1}{2}(a^2 - r_{n+1}^2)\theta}. \quad (5)$$

Next, we consider P_{R_n} and $P_{\tilde{R}_n}$ based on $P^*_{R_n}$ and $P^*_{\tilde{R}_n}$. P_{R_n} can be expressed as

$$\begin{aligned} P_{R_n} &= \sum_{m=1}^{\infty} \frac{e^{-\lambda R_n}(\lambda R_n)^m}{m!}(1 - (1 - P^*_{R_n})^m) \\ &= 1 - e^{-\lambda R_n P^*_{R_n}}. \end{aligned} \quad (6)$$

Here, we assume that there are m nodes that have received the packet and that $m \geq 1$. Because the nodes are uniformly distributed on a 2D plane follow a spatial Poisson distribution with parameter λ, the probability of m nodes in R_n is $(\lambda R_n)^m e^{-\lambda R_n}/m!$, where R_n is the area of the sector R_n. Similarly, $P_{\tilde{R}_n}$ can be expressed as:

$$P_{\tilde{R}_n} = \sum_{m=1}^{\infty} \frac{e^{-\lambda \tilde{R}_n}}{m!}(\lambda \tilde{R}_n)^m (1 - (1 - P^*_{\tilde{R}_n})^m) = 1 - e^{-\lambda \tilde{R}_n P^*_{\tilde{R}_n}}.$$

The next hop is located in R_n if a relaying node in R_n receives the packet but no nodes in $\tilde{R}_n$ do. Therefore, with P_{R_n} and $P_{\tilde{R}_n}$, the probability that the hop progress is in the interval (r_n, r_{n+1}) (i.e., $Pr\{r_n \leq Y_\lambda \leq r_{n+1}\}$) can be expressed as:

$$\begin{aligned} Pr\{r_n \leq Y_\lambda \leq r_{n+1}\} &= P_{R_n} \cdot (1 - P_{\tilde{R}_n}) \\ &= (1 - e^{-\lambda R_n P^*_{R_n}}) e^{-\lambda \tilde{R}_n P^*_{\tilde{R}_n}}. \end{aligned} \quad (7)$$

With (7), we can determine the expected hop distance in Theorem II.5.

Theorem II.5. *For a network with network density , the expected hop progress is expressed by* $E[Y_\lambda] \approx a - a e^{-\lambda \int_0^a Pt(y)\pi r dy}$

Proof: We first determine the CDF of the hop progress (i.e., $F_\lambda(y)$). Because $Pr\{r_n \leq Y_\lambda \leq r_{n+1}\} = F_\lambda(r_{n+1}) - F_\lambda(r_n)$ and $F_\lambda(0) = 0$, for any y in $(0, \infty)$, we can find an arc, for example r_n, to be infinitely close to y by dividing the semicircle into numerous infinitesimal arcs (i.e., $i \to \infty$). Therefore,

$$\begin{aligned} F_\lambda(y) &= Pr\{0 \leq Y_\lambda \leq y\} \\ &= Pr\{0 \leq Y_\lambda \leq r_1\} + Pr\{r_1 \leq Y_\lambda \leq r_2\} + ... \\ &+ Pr\{r_{n-1} \leq Y_\lambda \leq r_n = y\} \\ &= (1 - e^{-\lambda \int_0^\pi \int_0^y Pt(r) r dr d\theta}) e^{-\lambda \int_0^\pi \int_y^\infty Pt(r) r dr d\theta} \\ &= (1 - e^{-\lambda A(y)}) e^{-\lambda B(y)}. \end{aligned} \quad (8)$$

Here, for convenience of expression, we let $A(y) = \int_0^\pi \int_0^y Pt(r) r dr d\theta$ and $B(y) = \int_0^\pi \int_y^\infty Pt(r) r dr d\theta$. Note that, for OR transmission, the next-hop forwarders should be closer to the destination than the sender is; hence, the θ is set as π (i.e., 180°) to prevent the nodes farther from the destination from being selected as next-hop forwarders. Then, the PDF of Y_λ is:

$$\begin{aligned} f_\lambda(y) &= \frac{dF_\lambda(y)}{dy} \\ &= \lambda(A'(y) + B'(y)) e^{-\lambda(A(y)+B(y))} - \lambda B'(y) e^{-\lambda B(y)}. \end{aligned} \quad (9)$$

Because $Pt(r) = 0$ when $r > a$, random variable Y_λ takes only values in $(0, a)$, and the expected hop progress with network density λ, $E[Y_\lambda]$, can be calculated as

$$\begin{aligned} E[Y_\lambda] &= \int_0^a y f_\lambda(y) dy; \text{ that is,} \\ E[Y_\lambda] &= \int_0^a y[\lambda(A'(y) + B'(y)) e^{-\lambda(A(y)+B(y))} \\ &- \lambda B'(y) e^{-\lambda B(y)}] d \end{aligned}$$

$$\begin{aligned} Let\ u = y, dv = &[\lambda(A'(y) + B'(y)) e^{-\lambda(A(y)+B(y))} - \\ &\lambda B'(y) e^{-\lambda B(y)}] dy, \end{aligned}$$

which yields $v = e^{-\lambda B(y)} - e^{-\lambda(A(y)+B(y))}$. Therefore, we have

$$\begin{aligned} E[Y_\lambda] &= \int_0^a y f_\lambda(y) dy \\ &= y(e^{-\lambda B(y)} - e^{-\lambda(A(y)+B(y))}) \Big|_0^a \\ &- \int_0^a e^{-\lambda B(y)} - e^{-\lambda(A(y)+B(y))} dy. \end{aligned} \quad (10)$$

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where $y(e^{-\lambda B(y)} - e^{-\lambda(A(y)+B(y))})\Big|_0^a = a - ae^{-\lambda A(a)}$

and $\int_0^a e^{-\lambda B(y)} - e^{-\lambda(A(y)+B(y))}dy = \int_0^a e^{-\lambda B(y)}dy - \int_0^a e^{-\lambda(A(y)+B(y))}dy.$

Because $A(y) + B(y) = \int_0^\pi \int_0^a p(r)rdrd\theta = A(a)$, we can rewrite (10) as $\int_0^a e^{-\lambda B(y)}dy - ae^{-\lambda A(a)}$, thus the hop progress can be expressed by

$$\begin{aligned} E[Y_\lambda] &= a - ae^{-\lambda A(a)} - \int_0^a e^{-\lambda B(y)}dy + ae^{-\lambda A(a)} \\ &= a - \int_0^a e^{-\lambda B(y)}dy, \quad (11) \\ \text{where } B(y) &= \int_0^\pi \int_y^a Pt(r)rdrd\theta \end{aligned}$$

From (11), we find that $B(y)$ is not an elementary probability function. Thus, we approximate $E[Y_\lambda]$ by using a Taylor series. Here, we adopt the first-degree Taylor polynomial of $B(y)$ and express it as follows:

$$B(y) \approx B(0)/0! + B'(0)y/1! \text{ and}$$
$$B(y) = \int_0^\pi \int_y^a Pt(r)rdrd\theta = -\int_a^y Pt(r)r\pi dr$$

Because $Pt(r)r$ is continuous, the first part of the fundamental theorem of calculus gives

$$B'(y) = \frac{-d}{dy}\int_a^y Pt(r)r\pi dr = -Pt(y)y\pi.$$

Therefore, the expected hop progress with network density λ can be approximated as

$$E[y_\lambda] \approx a - ae^{-\lambda \int_0^a Pt(r)\pi rdr}.$$

■

2) Numbers of Transmissions with Network Density λ:

Corollary II.5.1. *For a network with network density λ, the average number of transmissions for each packet is $E[T_{s,d}^{MF/UT}] \approx D/a - ae^{-\lambda \int_0^a Pt(r)\pi rdr}$.*

Proof: The transmission propagation in MF/UT, as shown in Fig. 5, can be observed as a ripple with concentric circles from the source toward the destination. Note that nodes i and j may not really exist. They could be regarded as hypothetical averages of all the hop progressions in a transmission. In addition, the nodes are uniformly distributed in the network. Therefore, the average number of transmissions is of the same order as the required number of hops to be traversed. Accordingly, when the distance between any S-D pair in MF/UT is D, the average number of transmissions for each packet can be expressed using

$$E[T_{s,d}^{MF/UT}] \approx D/a - ae^{-\lambda \int_0^a Pt(r)\pi rdr}$$

■

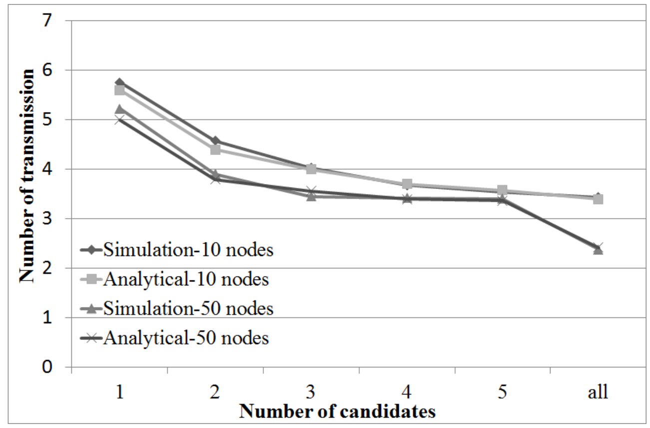

Fig. 6: Expected number of transmissions of SF/KT with different densities and k values

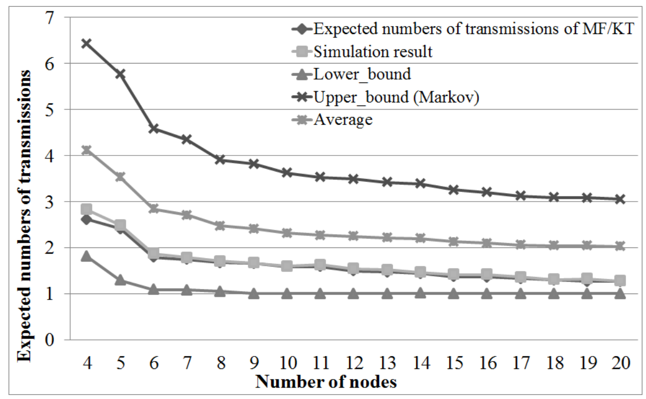

Fig. 7: Expected numbers of transmissions of MF/KT

3) Interplay of Network Density and Number of transmissions:

Corollary II.5.2. *For a network with density λ, the relationship of number of transmissions $E[T_{s,d}^{MF/UT}]$ and network density λ can be expressed as $\ln(aE[T_{s,d}^{MF/UT}] - D/aE[T_{s,d}^{MF/UT}]) \approx -\lambda \int_0^a Pt(r)rdr$.*

Proof: From Corollary II.5.1, we have $E[T_{s,d}^{MF/UT}] \approx D/a - ae^{-\lambda \int_0^a Pt(r)\pi rdr}$. After derivation, we can obtain $1 - (D/aE[T_{s,d}^{MF/UT}]) \approx e^{-\lambda \int_0^a Pt(r)\pi rdr}$.
By taking logarithms on both sides, we have $\ln(aE[T_{s,d}^{MF/UT}] - D/aE[T_{s,d}^{MF/UT}]) \approx -\lambda \int_0^a Pt(r)rdr$ ■

From this formula, we observe that the relationship between the number of transmissions and network density is logarithmic. Thus, we can evaluate the number of transmissions required from source to destination under a given network density.

III. Performance Evaluation

In this section, we will use our models to estimate the performance of OR networks in different scenarios and evaluate the accuracy of the three models through simulations. Because models in existing studies have different assumptions, settings, purposes and objectives, it is impossible for us to compare the

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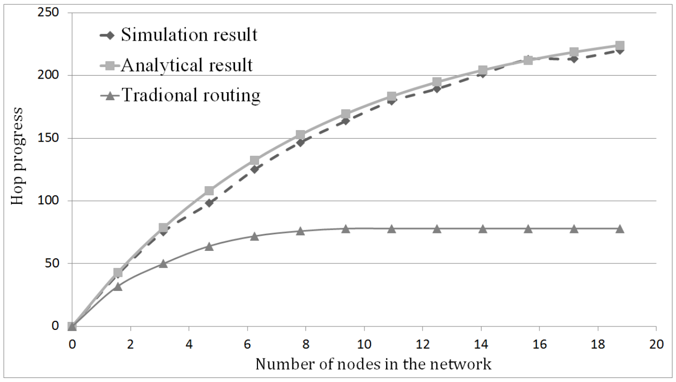

Fig. 8: Average hop progress per transmission under different numbers of nodes (lognormal shadowing model)

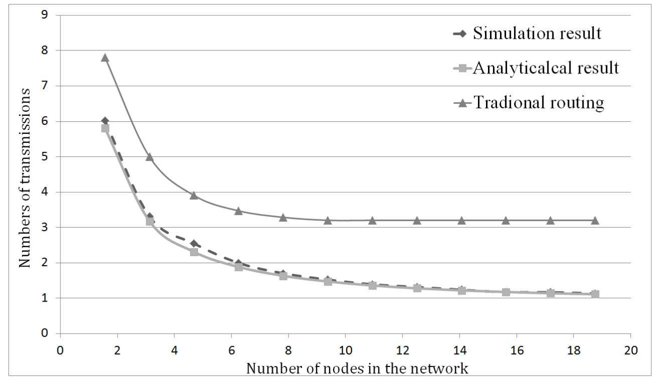

Fig. 10: Average numbers of transmissions under different numbers of nodes (lognormal shadowing model)

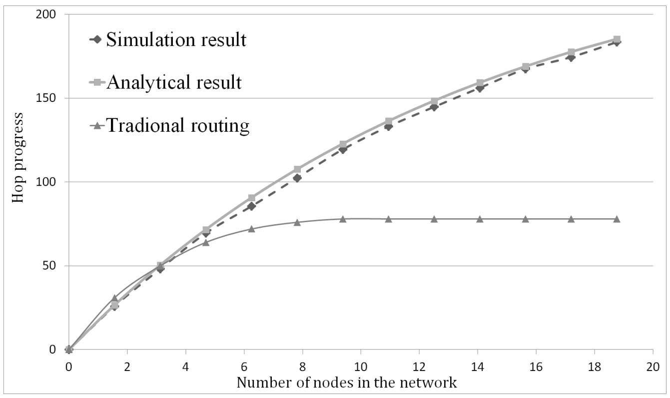

Fig. 9: Average hop progress per transmission under different numbers of nodes (Rayleigh fading model)

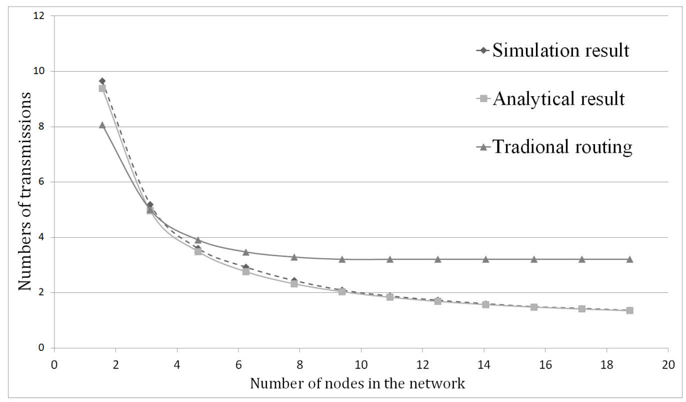

Fig. 11: Average numbers of transmissions under different numbers of node (Rayleigh fading model)

analytical results with those of other models. Therefore, we verify the analytical results with simulation results to evaluate accuracy. The settings are described in Section III.A, and the experimental results of SF/KT, MF/KT and MF/UT are then discussed in Sections III.B, III.C, and III.D, respectively.

A. Simulation Settings

The simulation results are based on the well-known lognormal shadowing models [36]. The packets are assumed to be decoded correctly only if the received SNR γ is greater than a given threshold Ψ. Therefore, the successful packet reception probability at a distance r is given by:

$$Pt(r) = Pt\{\gamma(r) > \Psi\} = \int_{\frac{\Psi P_{noise}}{P_{tx}}}^{\infty} \frac{e^{-\frac{1}{2}\left(\frac{\ln x - \ln(r^{-\alpha})}{\sigma}\right)^2}}{\sqrt{2\pi}\sigma x} dx.$$

where α is the path-loss exponent, σ is the standard deviation of the distribution of the received signal power, $\gamma(r)$ is the SNR received at distance r, P_{tx} and P_{noise} are the transmission power and noise, respectively. We use the default shadowing parameter settings of network simulator ns-2 [37]: α is set at 3.8, σ is set at 8.25 dB, and Ψ is 3.652×10^{-10} W, which is the typical operation range of IEEE 802.11n in outdoor environments. Note that since we would like to evaluate how network density affects delay in a network with a given density, the distance between source and destination in all the simulations are fixed at 250m to eliminate the impact of distance. Accordingly, the link between any two nodes exists only if the delivery probability is larger than 0.01. Therefore, the actual per-hop distance in our simulation results are always less than 250m and multiple hops are required in the simulation. The setting of the distance from source to destination will not affect the simulation results as long as such a distance (i.e., the distance between source and destination) is longer than the per-hop distance of each transmission. In fact, the packet delivery behavior will remain the same when the destination is not located within the transmission range of the source.

B. SF/KT

We first simulate the SF/KT model. The forwarder-selecting scheme used here is that in ExOR [1], because it is the most well-adapted scheme. We measure the results with two settings of network density N. Fig. 4 shows the expected number of transmissions of SF/KT (i.e., $E[T_{s,d}^{SF/KT}]$). Here, we vary the total number of nodes as well as the number of candidates (i.e., k). From Fig. 4, we observe that increasing k yields a reduction in delay. In other words, the chance to reach the destination increases as k grows. However, for low-density networks (i.e., 10 nodes), the gain is notable when k is below

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3; whereas, for high-density networks (i.e., 50 nodes), a further improvement can be achieved when k is larger. This means that a higher allowed number of forwarders can reduce delay in denser networks.

C. MF/KT

Fig. 5 shows the expected numbers of transmissions of MF/KT. The value of N varies from 4 to 20. We compare the numerical result of numbers of transmissions of MF/KT (i.e., $E[T_{s,d}^{MF/KT}]$ in Theorem II.2) with the two bounds. All scenarios (i.e., expected numbers of transmissions of MF/KT, upper and lower bounds) exhibit the same trend: the number of transmissions decreases as the number of nodes increases. The value of $E[T_{s,d}^{MF/KT}]$ is close to the result obtained through our simulations, which are bounded between the proposed upper and lower bounds. We also find that the average of the upper and lower bounds approximates the simulation result and is closer to the result than either the upper or the lower bound. Thus, this average value can be an accurate estimate for MF/KT scenarios.

D. MF/UT

Finally, we evaluate the MF/UT model. The node distribution is generated with a Poisson process. The network density, the average number of nodes in the network, ranges from 1.5625 to 16.75. Because this model does not require specific positions of nodes and only needs node density; therefore, for investigating the impact of large-scale fading (i.e., effect of obstacles) and small-scale fading (i.e., effect of multipath), we use two channel models in our simulated OR networks. In addition, we compare the performance difference between OR and traditional unicast best-path routing. The best-path routing uses Dijkstras shortest path algorithm where link weights are assigned based on the ETX metric. The accuracy under different fading conditions can also be verified because the analytical values are approximately equal to the simulation results in all cases. The two channel models are lognormal shadowing and Rayleigh fading, as discussed in [36]. By setting the fading power to 1, we can express the average SNR as $P_{tx}r^{-\alpha}/P_{noise}$. Hence, the probability that the packet is correctly received at distance r can be expressed as:

$$Pt(r) = Pt\left\{\gamma(r) > \Psi\right\} = e^{-r^{\alpha}\Psi P_{noise}/P_{tx}}$$

Both Figs. 6 and 7 show the average hop progress per transmission under different densities and fading models. The hop progress increases as the network density increases in both models. This is because the number of potential forwarders in the transmission range grows as the network density increases. We observe that the hop progress in the lognormal shadowing model is longer than that in the Rayleigh fading model because the more distant nodes in the lognormal shadowing model have superior delivery ratios. The average numbers of transmissions of MF/UT under two fading models are shown in Figs. 8 and 9, respectively. These two figures show that the average numbers of transmissions decrease as the network densities increase. This is because a larger density yields a larger hop progress, resulting in a lower delay. In Fig. 9, we observe that OR significantly outperforms traditional routing in terms of delay when the network density is high (i.e., network density > 5). This is because the packet delivery ratio under Rayleigh fading is very small when the inter-node distance is long, thus a high node density can improve performance greatly. Under both fading settings, our proposed models are very close to the results.

IV. Conclusion

In this paper, we have studied the numbers of transmissions of OR in multihop wireless ad hoc networks. Several models for OR protocols have been discussed in this paper. Our first model evaluates single forwarder OR under given knowledge of channel conditions. The second model analyzes multiple forwarder OR with known topology, whereas the final model finds the expected transmission delay of a multiple forwarder OR without known topology and is only based on node density. Several interesting results are observed in the simulations. First, limiting the candidate number causes different impacts under different densities in single forwarder OR. Second, for all three models, the lower the network density is, the longer the transmission duration. Finally and most importantly, our proposed methods can effectively estimate transmission delay under all three models.

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